MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Volume 119 No. 6 1959

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MEETING OF 1959 OCTOBER 9

Professor R. O. Redman, President, in the Chair

The President announced the death of William Hammond Wright, an Associate and Fellow of the Society, in tribute to whose memory the Fellows stood for a few moments in silence.

The election by the Council of the following Fellows was duly confirmed:—

Mark Gerard Landisman, Lamont Geological Observatory, Palisades, New York, U.S.A. (proposed by R. W. Girdler);

William Hedley Savage, Alrose, Stallingboro Road, Healing, Grimsby, Lincs. (proposed by W. T. Gayfer);

Shambu Dayal Sinvhal, Uttar Pradesh State Observatory, Naini Tal, India (proposed by M. K. Vainu Bappu);

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The election by the Council of the following Junior Members was duly confirmed :—

Roger A. Bell, Mount Stromlo Observatory, Mount Stromlo, Canberra, A.C.T., Australia (proposed by W. Buscombe); and

Philip Douglas Charles Heron Goodhart, Hertford College, Oxford (proposed by C. H. Barrow).

One hundred and ninety-five presents were announced as having been received since the last meeting, including :—

International Astronomical Union and International Scientific Radio Union: *Paris symposium on radio astronomy* (presented by the I.A.U.);

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Z. Kopal: *Close binary systems* (presented by Chapman and Hall Ltd.);

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S. Strizhevsky : *Nikolai Zhukovsky* (presented by Professor Kukarkin);

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D. Wattenberg: *Die Welt der Planeten* (presented by the National Book League);
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Several papers formerly belonging to the late Henry Perigal (presented by A. E. Perigal).

MEETING OF 1959 NOVEMBER 13

Professor R. O. Redman, President, in the Chair

The election by the Council of the following Fellows was duly confirmed :—

Charles Wilfred Earnshaw, The Old Cottage, The Square, Yarmouth, I.O.W. (proposed by H. P. Wilkins);
John Edward Colston Gliddon, Mill House, Chipping Hill, Witham, Essex (proposed by V. C. A. Ferraro);
Sydney Harold Hall, Imperial College, London, S.W.7 (proposed by R. G. Moon);
Adam B. Malone, B. P. House, Ropemaker Street, London, E.C.2 (proposed by D. T. Germain-Jones);
Edward Glyn Thomas, B.P. House, Ropemaker Street, London, E.C.2 (proposed by D. T. Germain-Jones); and
Margaret I. Watson, 2 Kew Terrace, Great Western Road, Glasgow, W.2 (proposed by T. R. Tannahill).

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Stephen Dresner, 70 Riverdene, Edgware, Middx. (proposed by R. H. Garstang);
Michael Friedjung, 33 Appledore Avenue, Ruislip, Middx. (proposed by P. A. Sweet);
Paul Geoffrey Murdin, 92 Northborough Road, London, S.W.16 (proposed by F. H. G. Best);
James Gordon Peters, Goethe Link Observatory, Indiana University, Bloomington, Indiana, U.S.A. (proposed by R. L. Sears);
Ian Walter Roxburgh, 6 Machon Bank, Sheffield 7 (proposed by R. v. d. R. Woolley); and
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MEETING OF 1959 DECEMBER 11

Dr. W. H. Steavenson, Vice-President, in the Chair

The Vice-President announced the death of William Moody Witchell, who had been a prominent Fellow of the Society for over forty-five years. The Fellows stood for a few moments in silence in tribute to Mr Witchell's memory.

The election by the Council of the following Fellows was duly confirmed :—

Habeeb Alvi, House No. III-C-3-994 Shah Gunj, Hyderabad Deccan, India (proposed by A. Ali);

Horace W. Babcock, Mount Wilson and Palomar Observatories, 813 Santa Barbara Street, Pasadena, California, U.S.A. (proposed by I. S. Bowen);

William Jack Baggaley, 44 Old Park Avenue, Sheffield 8 (proposed by N. S. Jinkinson);

Robert M. L. Baker, Jr., Department of Astronomy, University of California, Los Angeles 24, California, U.S.A. (proposed by S. Herrick);

William A. Baum, Mount Wilson and Palomar Observatories, Pasadena, California, U.S.A. (proposed by A. J. Deutsch);

William Morley Baxter, 164 Gunnersbury Avenue, Acton, London, W.3 (proposed by A. C. King);

Walter C. Beckmann, Lamont Geological Observatory, Palisades, New York, U.S.A. (proposed by R. W. Girdler);

Emilia P. Belserene, 9 Schley Avenue, New Rochelle, N.Y., U.S.A. (proposed by L. Motz);

Lyman H. Beman, 20336 Cantara, Canoga Park, California, U.S.A. (proposed by E. Burgess);

Irmgard Maria M. H. Berrer, 1 Well Street, Forsbrook, Stoke-on-Trent (proposed by E. Finlay-Freundlich);

George William Hobbes Berry, 6 Sunnymede Drive, Ilford, Essex (proposed by C. G. Saul);

Donald E. Billings, High Altitude Observatory, Boulder, Colorado, U.S.A. (proposed by W. O. Roberts);

Kenneth Bispham, 34 Winchester Road, Davyhulme, Urmston, Lancs. (proposed by S. W. R. Mottram);

Deane Robert Blackman, 31 Laver Street, Kensington 4, Victoria, Australia (proposed by R. L. Bryant);

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Harold H. Brown, 6 Earlspark Avenue, Glasgow, S.3 (proposed by D. Livingston);

Thomas Richard Carson, Theoretical Physics Division U.K., Atomic Energy Authority, Aldermaston, Berks. (proposed by D. W. N. Stibbs);

Subhash Chandra, Uttar Pradesh State Observatory, Naini Tal, India (proposed by M. K. Vainu Bappu);

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Robert Geoffrey Conway, 18 Granville Road, Wilmslow, Cheshire (proposed by J. G. Davies);

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William Edwin Fox, 49 Milner Street, Newark, Notts. (proposed by B. M. Peek);

Kenneth L. Franklin, American Museum-Hayden Planetarium, New York 24, U.S.A. (proposed by J. M. Chamberlain);

George David Garland, Geophysics Department, University of Alberta, Edmonton, Alberta, Canada (proposed by J. A. Jacobs);

Charles E. Gasteyer, Van Vleck Observatory, Wesleyan University, Middletown, Conn., U.S.A. (proposed by C. L. Stearns);

Kurt Gottlieb, Mount Stromlo Observatory, Research School of Physical Sciences, Mount Stromlo, Canberra, A.C.T., Australia (proposed by E. Gollnow);

David T. Griggs, Institute of Geophysics, University of California, Los Angeles 24, California, U.S.A. (proposed by J. C. Harrison);

Adelina Gutierrez, Casilla 36D, Santiago, Chile (proposed by F. Rutllant);

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John Hunter Reid, Dunsink Observatory, Castleknock, Co. Dublin, Eire (proposed by M. A. Ellison);

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Michael F. Roberts, Dept. of Exploration, B.P. House, Ropemaker Street, E.C.2 (proposed by D. T. Germain-Jones);

Ian Stewart Robertson, Bankhead Schoolhouse, Bucksburn, Aberdeenshire (proposed by R. V. Jones);

Andrew Arthur Neill Rule, 12 The Crescent, Surrey Hills, Melbourne, Australia (proposed by R. L. Bryant);

Netra Ballabh Sanwal, Uttar Pradesh State Observatory, Naini Tal, India (proposed by M. K. Vainu Bappu);

Peter August Georg Scheuer, Cavendish Laboratory, Free School Lane, Cambridge (proposed by F. Graham Smith);

Donald Charles Schmalberger, Goethe Link Observatory, Indiana University, Bloomington, Indiana, U.S.A. (proposed by R. L. Sears);

Branislav Sevarlić, Volgina 7, Belgrade, Yugoslavia (proposed by I. Atansijević);

Conrad H. Slater, Alpine Lodge, 95 Cambridge Road, Southport, Lancs. (proposed by A. C. B. Lovell);

René Simon, Institut d'Astrophysique, Université de Liège, Cointe-Sclessin, Belgium (proposed by P. Swings);

Roy Edward Rodney Smith, Jeffrey's Orchard, Afton Road, Freshwater Bay, I.O.W. (proposed by P. A. E. Stewart);

Richard Boynton Southworth, Harvard College Observatory, Cambridge 38, Mass., U.S.A. (proposed by T. E. Sterne);

Ranga Sreenivasan, Harvard College Observatory, Cambridge 38, Mass., U.S.A. (proposed by A. M. Naqvi);

Bernhard Sticker, Department of Astronomy, University of Bonn, Germany (proposed by W. Hartner);

Francis Michael Stienon, Harvard College Observatory, Cambridge 38, Mass., U.S.A. (proposed by C. P. Gaposchkin);

Henrietta H. Swope, 813 Santa Barbara Street, Pasadena, Cal., U.S.A. (proposed by I. S. Bowen);

William Tildesley, Electrical Research Association Laboratory, Cleeve Road, Leatherhead, Surrey (proposed by C. E. R. Bruce);

Malcolm M. Thomson, Dominion Observatory, Ottawa, Ont., Canada (proposed by C. S. Beals);

Kenneth Ulyatt, 134 Knight's Hill, London, S.E.27 (proposed by Lord Douglas of Barloch);

Mahendra Singh Vardya, Yale University Observatory, New Haven, Conn., U.S.A. (proposed by R. Wildt);

Raymond N. Watts, Jr., Robert T. Longway Planetarium, Flint, Mich., U.S.A. (proposed by C. H. Smiley);

Donald Ernest Weald, 23 Eden Road, Bexley, Kent (proposed by R. G. Foster);

Bengt Westerlund, Mount Stromlo Observatory, Canberra, A.C.T., Australia (proposed by W. Buscombe);

Lodewijk Woltjer, Sterrewacht, Leiden, Holland (proposed by S. Chandrasekhar); and George P. Woollard, Geology Department, University of Wisconsin, Madison, Wisconsin, U.S.A. (proposed by A. H. Cook).

The election by the Council of the following Junior Members was duly confirmed:—

David Michael Jordan, 16 Pembroke Avenue, Walton-on-Thames, Surrey (proposed by F. A. E. Pirani);

James Muirden, 90 West Cromwell Road, London, S.W.5 (proposed by P. Moore); and

Andrew Leslie James Wells, 43 Arlington Crescent, Waltham Cross, Herts. (proposed by F. A. E. Pirani).

Seventy-one presents were announced as having been received since the last meeting, including:—

International Astronomical Union: *The rotation of the Earth and atomic time standards*, Symposium No. 11 (presented by the I.A.U.);

M. Johnson: *Astronomy of stellar energy and decay* (presented by the author); and

D. W. Sciama: *The unity of the universe* (presented by the author).

THE DEFLECTION OF LIGHT BY THE GRAVITATIONAL FIELD OF THE SUN

George Darwin Lecture delivered by Professor A. A. Mikhailov on 1959 April 10

The great scientist Sir George Darwin investigated the action of gravitation on the Earth, manifesting itself in the tides, and also the figures of a rotating liquid mass and through them the cosmogonic consequences of gravitation. At his time there was hardly a question of an action of gravitation on light although this was foreshadowed in Newton's corpuscular theory of light. It was shown by Einstein that space around gravitating bodies is distorted, this causing a ray of light to travel in a curved path, the bending of the ray being double the amount that it would be if the ray consisted of material particles travelling with the speed of light and attracted according to Newton's law. In both cases, that is according to Newton as well as Einstein, the ray near a spherical gravitating mass would travel along an hyperbola, the mass being at its focus and the two asymptotes intersecting to form an angle α, which in the Newtonian case equals:

$$\alpha = 2\frac{fM}{c^2 r},$$

f being the constant of gravitation, M the mass, c the velocity of light and r the shortest distance of the ray from the centre of mass. It is easily seen that for all masses met with on the Earth, owing to the smallness of f and the greatness of c^2, this angle is exceedingly small—quite beyond any possibility of experimental detection. Only in the case of a mass of cosmical dimensions can this angle be of an observable value. The largest mass within the solar system is the mass of the Sun. Unfortunately, the least distance from its centre, at which a ray of light may pass, is its radius r; which being in the denominator of the above formula diminishes the angle α. Putting the corresponding values for the Sun we obtain: $\alpha = 0''{\cdot}875/r$ this being a very small angle but already observable. If Einstein's deduction is correct this angle should be doubled.

The angle $0''{\cdot}875$ gives the bending of a ray of light which just grazes the Sun's surface. Obviously the ray must come from a source situated beyond the Sun at a distance large enough for a substitution of the branch of the hyperbola by its asymptote. A star fulfills this condition. However a star at the very limb of the Sun's disk cannot be observed even during a solar eclipse: its light would be totally obliterated by the inner corona. Therefore r in our formula should be greater than the radius of the Sun. Expressing r in units of the Sun's radius we obtain $\alpha = A/r$

$$\text{in Newton's theory } A = 0''{\cdot}875, \text{ in Einstein's theory } A = 1''{\cdot}75. \qquad (1)$$

Now the coefficient in these formulae is proportional to fM/r and this is the potential of gravitation of a mass M for outer space. Thus the amount of deflection depends

on this potential. In the case of two adjacent parallel rays the difference of deflection, which amounts to the change in their angular distance, is proportional to the derivative of the potential, i.e. to the acceleration of attraction of the mass. This result will be useful for a discussion of other possibilities of tests besides observations during solar eclipses.

Now the verification by observation of the deflection as expressed by formula (1) presents two different problems. First of all there is the question of the law of deflection, that is the dependence of α on the distance r. According to theory, either Newton's or Einstein's, the deflection diminishes as $1/r$ with increasing r, that is hyperbolically. Logically speaking this question should be answered first. If this law is corroborated, then there arises the second question of the value of the constant A—whether it is consistent with the Newton or Einstein hypothesis, or is perhaps of a different amount.

I have just said that the sequence of verification is logically this: first to ascertain the law and then the amount. But not only logic, the history of science also shows us many examples of such a procedure. In our case the objection may be raised that the investigation of the deflection of light does not necessitate the establishment of a new principle, as the law of deflection is deduced from more general laws and therefore there should be no doubt about the law itself and only the amount of deflection has to be decided upon. As Eddington, in his inspiring book *Space, Time and Gravitation,* stated: "There were two questions to answer: firstly whether light has weight (as suggested by Newton) or is indifferent to gravitation; secondly, if it has weight, is the amount of deflection in accordance with Einstein's or Newton's laws ?" Nevertheless when testing the deductions of theory by experiment or observation the investigation should be made as thoroughly and as many-sided as possible. It would be imprudent to accept *a priori* part of the deduction and to build the investigation of the remaining part upon this assumption.

Sometimes both problems can be solved simultaneously and the observations can be used for checking the law and determination of the constants involved. However the quality of the observations for this purpose must be very high. Unfortunately this is not the case for the Einstein shift. So far only the value of the constant A in formula (1) has been sought without any success in verifying the formula itself. The reason for this is the extreme difficulty of the problem from an astrometrical point of view. It must be borne in mind that, usually, the stars nearest to the Sun that can be observed during a total eclipse are at least two semidiameters of the Sun's disk from its centre, so that the maximum shift for such stars amounts to only half the value of the constant A. It is quite hopeless to determine the positions of such stars from observations and compare them with those taken from a catalogue, as the precision of such a procedure would be quite insufficient. The star catalogues contain errors of several tenths of a second of arc in the star coordinates even for the epoch of observation of the catalogue, which in most cases differs by many years from the year of observation of the eclipse. Even in the more favourable case of Einstein's value of the constant it would amount to $0''\cdot87$, which corresponds (for an average focal length of the observing telescope of 6 m = 20 ft) to about 0·025 mm in the focal plane. It is obvious that the observations must be made with the aid of photography owing to the short duration of total eclipses of the Sun. This ensures only a determination of relative positions of stars, which is quite sufficient for the purpose. These positions must be

compared with those determined with the same instrument in night conditions, i.e. when the Sun is far away from the eclipse field of stars, some six months before or after the eclipse and when the stars occupy their normal unshifted places.

Thus there would be at least two plates or a number of pairs of plates to be compared between themselves. Each pair would consist of one "eclipse plate" and one "night" or comparison plate. It would be of advantage to have the comparison plates taken "through the glass" in order that it be possible to make all the measurements differentially by putting the two plates together, using the film-to-film method, and measuring only the small distances between the corresponding images, usually amounting to fractions of a millimetre. But the night or comparison plate would be taken at a different temperature than the eclipse plate and this alone would cause a change of scale of the photographic image; besides the instrument would be in different conditions: it may have been dismounted during the interval, the hour angle may be different, and so on. Everyone acquainted with photographic astrometry knows how all these details affect the precision of the results. However if the scale is not quite identical in the two photographs, formula (1) must be supplemented by the addition of a term which takes account of this circumstance, i.e. it should be written : $\alpha = A/r + Br$, where B is the coefficient of the correction for the difference of scale. The angle α causes a shift of the stars in a radial direction from the centre of the Sun by the amount Δr, which is determined by measuring the plates. Thus the equation of condition for each star must be written in the form :

$$\Delta r = \frac{A}{r} + Br, \tag{2}$$

the two constants A and B to be determined by a least-squares solution and Δr expressed in seconds of arc.

In order to get a reliable determination of both constants it is necessary to have a wide range in the distance r. In this case the constant B, which is not of interest to us, but has perforce to be derived for the determination of A, is obtained mainly from the stars with large values of r, as for them the corresponding term in our equation is also large. The constant A, on the other hand, is then determined by stars with small values of r, i.e. situated as near the Sun's limb as can be observed. Usually however the smallest value of r for such stars, expressed in units of the Sun's radius, is about 2, as the nearer stars are obliterated by the light of the corona. The largest values of r should then be not less than 6 or even 8, which requires a field of view of about $4^\circ \times 4^\circ$. The focal length of the camera must be sufficiently long to allow a precise determination of the star positions, i.e. not less than 3 metres. In this case the size of the plates would be much larger than 20×20 cm. The handling and rapid replacement of plates of such size during the short duration of the total eclipse would not be an easy matter. It is also difficult to mount such a large instrument equatorially, especially in the conditions of an expedition, and therefore a coelostat is often used. The latter however involves the danger of deformation of the mirror due to the cooling of the air during the eclipse.

Such are the main difficulties encountered in observing the Einstein effect during total solar eclipses and it is due to them that the results so far are not of a very satisfactory nature. We will now proceed with a review of the observational data.

The first attempt to observe the deflection of light from the stars on Newton's principle was planned by an American and a German expedition to Russia to observe the eclipse of 1914 August 21, but bad weather and the outbreak of war frustrated this intention. Meanwhile, in 1915, Einstein deduced his double value of deflection and the question arose of a discrimination between the two theories. As soon as the war conditions made it possible, the Royal Society jointly with the Royal Astronomical Society formed a committee for making the necessary preparations for the organization of observations. Fortunately there was a very exceptional eclipse forthcoming on 1919 May 29, when the eclipsed Sun was to be projected on a very favourable background of stars: the well-known group of the Hyades in the constellation of Taurus. On this date about one dozen stars of unusual brightness, between the 4·5 and 6·0 stellar magnitudes, were located near the Sun. The path of totality of this eclipse passed from South America over the Atlantic Ocean and through equatorial Africa, ending in the Mozambique Channel near Madagascar. Two British Expeditions were sent, one headed by Eddington to Principe, a small island in the Gulf of Guinea, the other with Dr Crommelin and Dr Davidson to Sobral in North Brazil. Both were equipped with long-focus astrographic cameras fastened horizontally with coelostats in front of the lenses to reflect the rays of the stars in a fixed direction into the corresponding camera. In Principe the weather conditions were unfavourable and the plates were exposed on a partly cloudy sky. On one of the photographs five stars could be measured, their positions on the eclipse plate being compared with those on plates taken with the same instrument four months previously in England in night conditions, when the Sun was far from the eclipse field. The results, although not very reliable, were in favour of the Einstein value of deflection. The expedition to Brazil was more fortunate and with the longer of the two cameras, having a focal length of 19 ft (5·7 m) and 4 inch aperture, seven plates were obtained with good images of 7 stars on each plate. The observers and instruments remained at the place of observation for two months and the comparison plates were taken through the glass to enable differential micrometric measurements of the star images. The constant of deflection, as given by the observers, is $A = 1''\cdot98$. It must be pointed out that this achievement is quite outstanding and a very fine performance in photographic astrometry.

Dyson and Woolley, in their book *Eclipses of the Sun and Moon* (Oxford 1937), give a revised table of the observed deflections after correcting for second-order terms of refraction. I used these deflections for forming the equations of condition (2), which were solved by the method of least-squares, and obtained $A = 2''\cdot07 \pm 0''\cdot09$ p.e. and the scale correction $B = -0''\cdot0158$ for a distance of one radius of the Sun's disk. In order to compare this result with the theoretical deflection, another solution was made, inserting in equation (2) the value $1''\cdot75$ for A and determining the corresponding scale correction, which in this case of course differs from the previous value. I obtained $B = +0''\cdot0068$ and it is obvious that the positive value of B must help the deficiency of A to build up the high values of the observed Δr.

Yet another solution was attempted, representing the observed deflections by a straight line of the form: $\Delta r = a + br$. This formula has no theoretical foundation whatever and is purely interpolational for a restricted range of r, as it does not hold for large values of r. A special scale correction is not necessary in this case, as it is included in the constant b. If a were zero, then the whole observed

deflections could be explained by a change of scale between the eclipse and comparison plates. The following expression was found: $\Delta r = 1''\cdot 366 - 0''\cdot 208\,r$.

Thus we have three solutions: the first giving the value of A which fits the observations best, the second for the theoretical value $A = 1''\cdot 75$ and the third the straight line solution. Their relative reliability can be judged by the sum Σv^2, v being the residuals of the corresponding equations of condition: (1) $\Delta r = A/r + Br$, (2) $\Delta r = 1''\cdot 75/r + Br$, (3) $\Delta r = a + br$. In our case, if v is expressed in units $0''\cdot 01$, we have for the respective solutions: (1) $\Sigma v^2 = 360$, (2) $\Sigma v^2 = 690$, (3) $\Sigma v^2 = 402$. Thus it is quite certain that the observed deflection is larger than Einstein's value if the theoretical hyperbolic law of inverse distance is assumed.

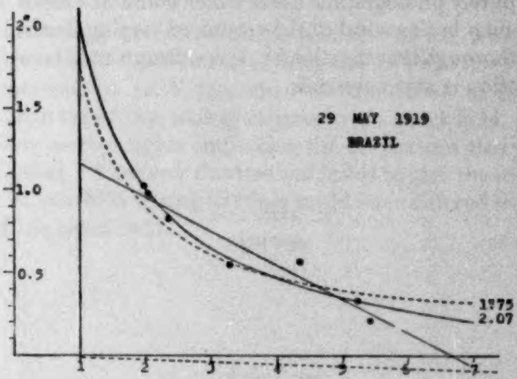

FIG. 1

All three solutions are shown in Fig. 1, which needs some explanation*. The distances from the Sun's centre in units of the radius of the solar disk are plotted along the horizontal axis. The black dots show the observed deflections of the individual stars counted vertically from the horizontal axis. The calculated deflections are shown by the solid hyperbolic curve for $A = 2''\cdot 07$ and the dotted curve for $A = 1''\cdot 75$. The ordinates of the former are reckoned not from the horizontal axis but from the inclined straight line which represents the scale correction, i.e. the term Br of formula (2). The curve for $A = 1''\cdot 75$ has a different scale correction which is not shown on the diagram. The three nearest stars to the Sun lie very nearly on the theoretical curve for $A = 2''\cdot 07$ but, if the lines drawn in the diagram are not taken into consideration and attention is paid only to the seven stars, the straight line seems to fit best. If the large observed value of A is real and, as we shall presently see, an analysis of subsequent observations seems to corroborate this, then the question arises of the origin of this discrepancy with the theory of relativity.

First of all there is the possibility of a refraction of light in the outer solar atmosphere—the corona—which would act in the necessary direction. However it can easily be shown that to explain an additional deflection, which increases the

* It must be noted that all six diagrams showing the results of the different observations are drawn on the same scale and are therefore strictly comparable.

constant A by only $0''{\cdot}1$, an inadmissibly large density of the corona must be assumed. The total optical path of a ray of light traversing such a dense corona would be many hundreds and even thousands of times greater than the optical path through a uniform atmosphere of the Earth, which absorbs about 0·8 stellar magnitudes. As the absorption of light expressed in magnitudes is proportional to the thickness of the absorbing medium, it would be quite impossible to see the stars, even through the outer parts of the solar corona, and no stars would be shown on the eclipse photographs. This fact seems to be quite conclusive.

Then it was conjectured that the one-sided cooling of the Earth's atmosphere during totality may cause refraction anomalies, but this seems to be highly improbable as the various eclipses were observed under very different climatic conditions. A purely photographic effect which could be caused by the images of the stars falling on a background of the corona of varying density was suspected. However a very thorough investigation by A. von Brunn and H. von Klüber showed that this explanation is also untenable.

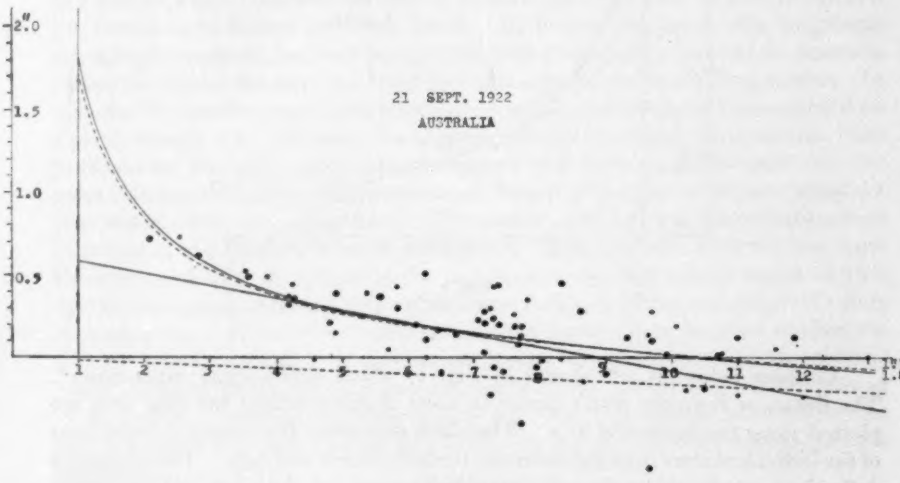

Fig. 2

The next successful attempt was made by an expedition of the Lick Observatory at Wallal on the north-western coast of Australia. A twin camera of 4·5 m focal-length and 5" aperture was used. The plates were of the size 17 inch × 17 inch, covering $5°{\cdot}4 \times 5°{\cdot}4$, ¼ inch thick and weighed 6½ lbs each. Four good photographs were obtained with 92 stars to the 10·5 magnitude. The comparison plates were taken four months earlier at Tahiti, the instrument being dismounted during this interval. To avoid a possible distortion caused by a coelostat the astrograph was mounted equatorially and pointed directly towards the star field. Weights were attributed to the individual stars according to the quality of the image as seen in the measuring machine, those of worst quality being discarded. In the different combinations of the eclipse and comparison plates from 62 to 85 stars could be measured. The final result as given by the observers is $A = 1''{\cdot}72 \pm 0''{\cdot}11$. I have made a new computation using the data published in a very detailed article in the Lick Observatory Bulletin. A few stars with very diffuse images were

rejected; 71 stars were kept and assigned less varied weights. The range of r is very large: from 2·1 to 11·0. My result is $A = 1''\!\cdot\!83 \pm 0''\!\cdot\!11$ p.e. The probable error of one star of weight 1 is $\pm 0''\!\cdot\!13$. Fig. 2 shows the deflection for the individual stars, the size of the dots giving their weight. An inspection of this graph shows that only three stars nearest to the Sun give an upward bend to the curve, otherwise no curvature is noticeable. However, even if these stars are kept a straight line solution fits the observed deflections best. The following sums of squares of the residuals have been obtained:

solution for $A = 1''\!\cdot\!83$	$\Sigma v^2 = 425$
$A = 1''\!\cdot\!75$	446
straight line	419

At this time it was fully realized that the necessity of the determination of a scale difference takes much weight from the constant A and that it is very important to devise some method for an independent determination of this difference. Already the Australian expedition took photographs of a check field, more than 90° distant from the Sun, on two nights embracing the eclipse and also photographed this same field at Tahiti. However this method failed to give the required results as the scale of the eclipse field during daytime could have differed from that of the night exposures of the check field.

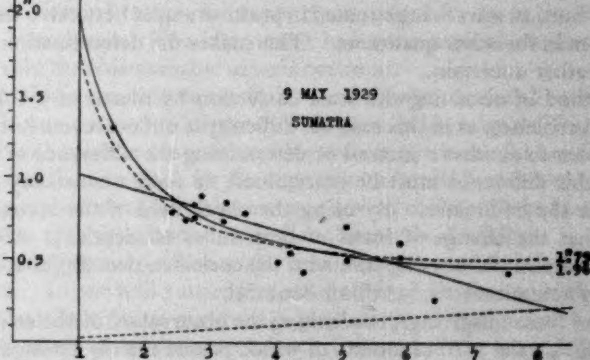

FIG. 3

In view of this, very elaborate preparations were made at the Potsdam Astrophysical Observatory for the observation of the eclipse of 1929 May 9 in North Sumatra (Fig. 3). A special instrument was constructed for this purpose. It consisted of two identical photographic cameras of 8·5 m focal length and 20 cm aperture, fixed horizontally in two azimuths differing by about 25°. Both cameras were fed by one and the same coelostat so that the two star fields, with a distance of about 25° between them, could be photographed simultaneously. The plates, 45 × 45 cm, covered 3° × 3°. When one of the cameras was pointed at the eclipsed Sun, the other served for photographing a check field. Besides, a collimator with a rectangular grating in its focal plane and directed towards the coelostat mirror was used for printing a standard scale on the plates. This standard scale also served for an investigation of the distortion of the film. Along with this equipment an

equatorially-mounted wide-field astrograph of 3·4 m focal length was used; this was pointed alternately, first at the eclipse field, and then, without a change of the plate, at the check field distant from the Sun. The latter field was destined for a determination of the scale. However this method did not give reliable results, possibly owing to the different flexure of the instrument.

The authors, Professor Freundlich, von Klüber and von Brunn, in a series of very detailed articles thoroughly discussed their observations from many sides. As a final result, from four plates taken with the horizontal camera, they give $A = 2''\cdot 24 \pm 0''\cdot 10$, the scale correction being determined by means of the collimator. I have derived a solution according to equation (2) in the usual way, i.e. determining the scale difference from the measurements of the stars using the comparison plates, and found $A = 1''\cdot 96 \pm 0''\cdot 08$, $B = -0''\cdot 016$, the p.e. of one star being $\pm 0''\cdot 15$. The sum of squares for the three solutions is as follows:

solution for $A = 1''\cdot 96$	$\Sigma v^2 = 1971$
$A = 1''\cdot 75$	2372
straight line	3236

This time we see that there is a marked inferiority of the straight line solution, chiefly due to the influence of a single star, which is nearest to the Sun's disk, with $r = 1\cdot 52$. If this star is excluded the respective Σv^2 are 1962, 2291, 2011. Moreover it must be noted that the distribution of stars was very unsymmetrical around the eclipsed Sun, 16 stars being situated in position angles between 0° and 180° and only two stars in the other quadrants. This makes the determination of the scale constant B rather uncertain.

The method of obtaining the scale correction by means of a collimator has met justified criticism, as in this case the difficulty is not overcome but transferred from one place to another: instead of determining the difference of scale of the astrograph this difference must be determined, or even a constancy of the scale assumed, for the collimator. By using the check field of the second tube it is stipulated that the change of focus of both tubes is identical. Moreover the German expedition did not dispense with the coelostat, deeming that it could not give rise to systematic errors, but this is doubtful.

In view of these misgivings, in planning the observation of the eclipse of 1936 June 19, (Fig. 4), the path of totality of which passed mainly through the USSR from the Black Sea to the Pacific Ocean, I designed a method which seemed to guarantee a good and independent determination of the difference of scale between the eclipse and comparison field of stars.

For this purpose a plane-parallel glass of the finest optical quality is placed in front of the object glass of the astrograph at an angle of about 45° to the optical axis. This plate reflects by both sides about 9 per cent of the light coming from a check star field some 90° distant from the eclipsed Sun. In this way, during the exposure of the eclipse field, a check field is photographed simultaneously on the same plate, the stars of the latter field showing fainter by 2·5 magnitudes. The star field for this eclipse was satisfactory, having a number of stars of the 8th and 9th magnitude. So that the stars of the check field be of about the same brightness their true magnitude should be between 5·5 and 7. For this purpose the group of Coma Berenices served admirably. Selecting Kouybyshevka, which is in the Far East and on the Trans-Siberian railroad, as an observation site, the altitude of

this group during the eclipse could be nearly identical with the altitude of the eclipse field, i.e. 36° above the horizon. The method consists in the following procedure: during the eclipse the eclipse field is photographed simultaneously with the check field. About six months later (or earlier) these same fields are photographed at the same hour angle through the glass side of a photographic plate, thereby furnishing the necessary comparison plate. The check field on the two plates, not being affected by the Einstein displacement, serves then for a determination of the scale correction. In order to lessen the accidental errors, especially those due to the distortion of the film, several photographs should be taken during the eclipse (as many as possible, the number depending on the duration of totality) and at least the same number for comparison after or before, with an interval of six months.

Our instrument had a 15 cm diameter object glass, the focal length being 600 cm. The plane-parallel glass plate, 27 cm in diameter and 3 cm thick, was made at the Optical Institute in Leningrad by Maksutov and is of the highest quality, the wedge being less than 1″. The photographic plates, 35 × 35 cm, covered 3°·3 × 3°·3 and were of plate glass 6 mm thick. They were specially made by Ilford Ltd. for this purpose and coated with an unsensitized "Zenith" emulsion. Each plate weighed nearly 2 kg and the construction of plate holders for such large and heavy plates, ensuring the required precision of the position of the plate in relation to the lens, is not simple. Moreover the handling and quick change of plate holders of such dimensions and weight during the eclipse would be very difficult and a prolonged length of time between the exposures required for the extinction of vibrations of the instrument. The instrument had to be pointed directly at the Sun in order to avoid the objectionable use of a coelostat.

In view of this circumstance a quite unusual construction of the astrograph was adopted. It was decided to dispense with plate holders by making the camera large enough to house the observer and his assistant, who could handle the plates in darkness, put them in place for the exposure and quickly remove them, making the necessary replacement. In this manner it was possible to take four plates with exposures from 25 to 35 seconds during the 138 seconds of totality.

A light-tight plywood pavilion with a base of 4 × 5 m served as the outer covering of the camera. In one wall a stone pier was built in. On the top of this stone pier was mounted an iron frame carrying in gimbals the upper end of a framework, 6 m in length, made of welded iron tubes. One end of the latter held the object glass, the other a triangular plate with three screws, on the pointed ends of which rested the photographic plate. The lower end of the framework glided along a steel cylinder, being drawn by an attachment to the nut of a screw which was turned by clockwork. Several counterweights on steel cables passing over pulleys ensured easy gliding.

During the eclipse the observer put the photographic plate in position on the ends of the screws and operated the shutter. Unfortunately of the four plates taken during the eclipse only two could be used for measurements. The first plate was spoilt by opening the shutter too soon and another plate had to be discarded as it did not rest properly on the three screws.

The comparison plates could be taken only during the following winter which was unfortunate as the temperature then in eastern Siberia is very low. In order to avoid the greatest frost the time was chosen as near to spring as possible and the comparison plates were taken in March 1937. Nevertheless, the temperature was

then $-21°C$. During the eclipse the temperature was $+23°{\cdot}6\,C$. This great difference in temperature caused a large difference in the scale between the eclipse and the comparison plates, which had to be determined in the usual way, i.e. by using equations (2). The cause of the failure of the use of the check field for the determination of the scale is probably the following. The movement of the astrograph, which was determined by having the right direction of the guiding cylinder and the speed of the driving clockwork, was adjusted so that the eclipse field remained stationary on the photographic plate during the exposure. As is known, if in an equatorial mounting the hour axis is not exactly parallel to the earth's axis, then by proper guiding a star can be kept stationary in the field of view but the whole field rotates around the star used for guiding. In our case the farthest star of the eclipse field was $1°{\cdot}9$ from the centre of the plate, which was guided correctly. The check field was about 98° distant. If the image of the sky rotated around the Sun this would cause a linear displacement of the image of a star proportional to the sine of its angular distance from the Sun or $\sin 98° : \sin 1°{\cdot}9 = 30 : 1$ for stars in the check field to the farthest star in the eclipse field. Thus the images of the stars of the check field could be drawn out to such an extent that they would not be discernible at all, while the stars of the eclipse field would be quite sharp. Probably this happened in our case and this is the reason why the check field could not be used for a determination of the scale. In the future equal attention should be paid to the adjustment of the driving according to both fields.

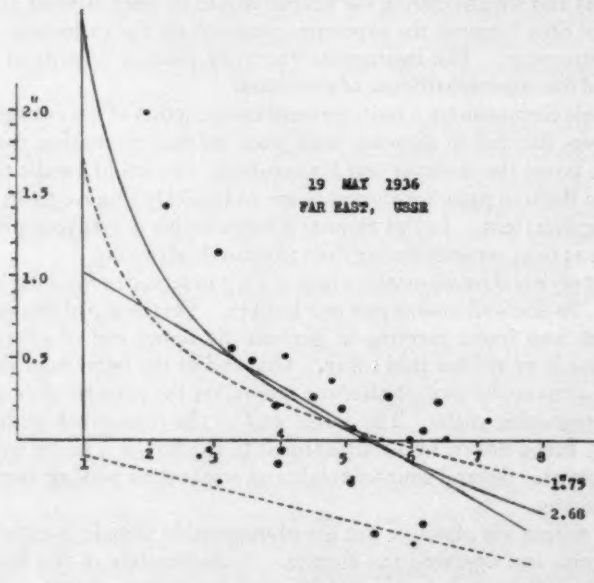

FIG. 4

Using two comparison plates taken through glass four different combinations with the two eclipse plates were possible. On the whole 29 stars could be measured but not in all the combinations as some of them were too faint on one of the plates. Weights were assigned according to the number of measured images. A least squares solution of the equations of condition gave an unusually

large value for $A = 2''{\cdot}68 \pm 0''{\cdot}37$ and the sum of the squares of the residuals for the different solutions:

$A = 2''{\cdot}68$	$\Sigma v^2 = 1375$
$A = 1''{\cdot}75$	1534
straight line	1637

Now I must confess that after obtaining the value of deflection from a complicated calculation one has no great conviction in the reality of an apparent repulsion of the stars from the Sun at all, as this is not very evident from the measurements themselves. However the following considerations gave me a very obvious proof of the reality, if not of the derived value of A of at least the existence of a noticeable repulsion of the theoretical order of magnitude. In the measuring machine the combined plates were measured differentially, the differences of the rectangular coordinates, Δx and Δy, between the stars on the eclipse and comparison plates being found. The coordinate axes were orientated tangentially to the declination circle and the diurnal parallel passed through the centre of the plates. The radial displacement Δr to be inserted in the equations of condition is in this case: $\Delta r = \Delta x \sin P + \Delta y \cos P$, where P is the position angle of a star referred to the Sun's centre. These Δr evidently contain the Einstein deflection. But a tangential component can also be calculated from : $\Delta s = \Delta x \cos P - \Delta y \sin P$, which does not contain any deflection at all. Now the sum $\Sigma(\Delta r)^2$ was 2932, which after a subtraction of the deflection diminished to 1375 with a p.e. of one star of weight $1 = \pm 0''{\cdot}50$. The corresponding sum for the tangential component was 1705 with a p.e. $\pm 0''{\cdot}52$ or practically identical with that of the radial components freed from the deflection of light and this is proof of its reality.

Since 1936 the instrument has been improved by replacing the gliding movement along the cylinder by a rolling movement on a plane surface which has to be orientated parallel to the equator. However I had no luck at the eclipse of 1941 September 21, as the war conditions frustrated observations at Alma-Ata, although the weather was favourable. On 1945 July 9, at Rybinsk on the Volga, as well as on 1947 May 20, in Brazil and on 1954 June 30, in the northern Caucasus the sky was overcast.

In November 1946 during a visit to the Yerkes Observatory I learnt from Dr van Biesbroek that he had devised a similar method of observing the deflection of light which he intended to apply at the eclipse of 1947 May 20, in Brazil. Dr van Biesbroek's instrument had an equatorial mounting of a more conventional type, but the plane-parallel glass in front of the object-lens was silvered so as to transmit and reflect nearly the same quantity of light. I think that this is not as good as an unsilvered plate because much of the light of the most valuable eclipse field is lost. As regards to the check field it is always possible to select a field with a sufficient number of bright stars, if the condition of a distance of 90° from the Sun is not strictly maintained. If 50 per cent of the light from the eclipse field is lost the exposure time must be doubled, which diminishes the number of plates that can be taken during totality, thereby reducing the precision of the result.

It was due to this circumstance that at the eclipse of 1947 May 20, Dr van Biesbroek took only one plate with an excessive exposure of 185 seconds. This was thought necessary in view of the intention to use a yellow filter. However, the filter had to be removed owing to its deterioration shortly before the eclipse, but the time of exposure was not shortened. As a result the plate was strongly fogged and the nearest star that could be measured was 3·3 radii from the Sun's centre.

Dr van Biesbroek writes in his article in the *Astronomical Journal** that the stars of the check field were astigmatically distorted, which he ascribed to a deformation of the plane-parallel glass because of unequal cooling during the eclipse. I suspect that at least partly this was caused by the above-mentioned fault in the guiding of the telescope. As the check field could not be used, the determination of the scale correction was made in the usual manner by means of equations (2). These were however transformed according to a proposal by Professor Danjon, i.e. both parts were divided by r, which gives:

$$\frac{\Delta r}{r} = \frac{A}{r^2} + B. \tag{3}$$

This procedure was recommended in order that the solution by the method of least squares be simpler. However it must be remembered that the multiplication or division of the equations of condition by any factor is equivalent to an alternation of the weight. Thus by dividing by r, the weights of the nearest stars to the Sun's disk are unduly increased. The natural distribution of weights is given only by the original form of equations (2). The complication arising in solving these equations in comparison with equation (3) is quite negligible as the labour involved in the solution of equations of any form is very small in comparison to the labour connected with the expedition and observations.

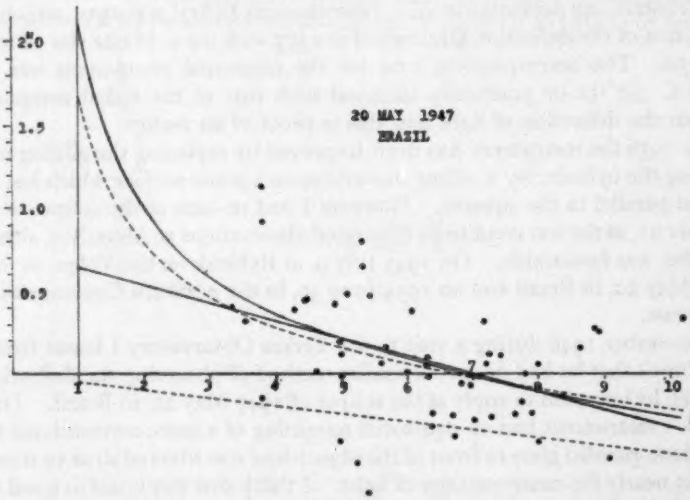

Fig. 5

The author gives for A, as determined by the method proposed by Danjon, $A = 2''{\cdot}01 \pm 0''{\cdot}18$ p.e. We have recomputed the observations as published in the *A.J.*† according to the usual formula (2) with the result $A = 2''{\cdot}20 \pm 0''{\cdot}38$. Using Danjon's transformation we found $A = 2''{\cdot}41$ and it is an enigma to us how van Biesbroek obtained his results. It is almost obvious that the solution as proposed by Danjon must be larger than the usual one as an inspection of Fig. 5 shows that the nearest stars, which in this case have the greatest weight, demand an increase in the value of A.

* *A.J.*, **55**, No. 1182, p. 49, 1950.

† In this article there are several misprints. The values of X for the stars 43 and 44 must be interchanged, also for stars 34 and 35; and those for stars 14, 18 and 26 are wrong.

The sum of the residual squares for the three solutions is:

$A = 2''\cdot 20$	$\Sigma v^2 = 612$
$A = 1''\cdot 75$	618
straight line	630

We see that there is very little difference between these numbers so that the best solution is very uncertain, this being due to a large spread in the observed deflections of the individual stars as well as to the absence of stars near the Sun's limb.

And now we come to the last, and in one respect the most successful, determination of the deflection of light. During the eclipse of 1952 February 25 in Khartoum, van Biesbroek took two plates with the same instrument that he used in 1947. The exposures of 50 and 90 seconds showed respectively 9 and 11 stars in the eclipse field and 8 stars in the check field, the star images being diffuse due to a gusty wind that shook the instrument. Two comparison plates were taken six months later, the instrument remaining undisturbed on the site during this time. The check field could be used for a determination of the difference of scale, and the four combinations of the eclipse and comparison plates were measured twice. The observer's result as printed in the *Astronomical Journal** is $A = 1''\cdot 70 \pm 0''\cdot 07$ p.e. There is no indication in the very laconic article published whether the method proposed by Danjon was used for the solution.

We have recalculated the measures as published and found that no additional scale correction is required, so that the method of observation has justified itself. However we obtained quite a different value for $A = 1''\cdot 43 \pm 0''\cdot 18$ p.e. That van Biesbroek's value of A does not fit his measurements is shown by the residuals, the algebraic sum of which with a consideration of weights is for:

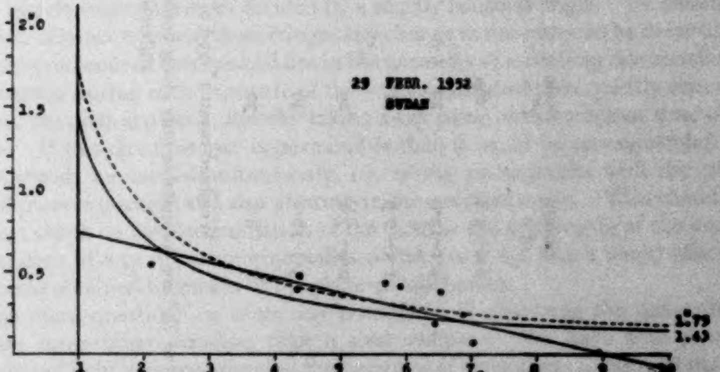

FIG. 6

$A = 1''\cdot 43$	$\Sigma pv = +0''\cdot 02$	$\Sigma pv^2 = 71$
$A = 1''\cdot 70$	$-0''\cdot 94$	82
straight line		40

The sum pv^2 also shows very clearly that the value $1''\cdot 70$ for A is quite inadmissible. In this case the straight line solution is best. An inspection of Fig. 6 shows that the nearest star, with $r = 2\cdot 12$ gives a comparatively small deflection. This star pulls the whole hyperbolic curve downwards and this explains the small value

* *A.J.*, 58, No. 1207, p. 87, 1953.

TABLE I

Year	Number of plates	Number of stars	Limits of r	Scale $1''=$	p.e. of one star	A as given by observer	A as given by new reduction	Σv^2 A	Σv^2 $1''\cdot75$	Σv^2 straight line
1919	7	7	2·0– 5·4	28μ	$0''\cdot15$	$1''\cdot98$	$2''\cdot07 \pm 0''\cdot09$	360	690	402
1922	4	71	2·1–13·0	22	0·13	1·72	$1\cdot83 \pm 0\cdot11$	425	446	419
1929	2	18	1·5– 7·5	41	0·15	2·24	$1\cdot96 \pm 0\cdot08$	1971	2372	3236
1936	2	29	2·0– 7·2	29	0·27	2·70	$2\cdot68 \pm 0\cdot37$	1375	1534	1630
1947	1	51	3·3–10·2	30	0·24	2·01	$2\cdot20 \pm 0\cdot38$	612	618	630
1952	2	11	2·1– 8·9	30	0·15	1·70	$1\cdot43 \pm 0\cdot18$	7058	8693	4039

Weighted mean $1''\cdot93 \pm 0''\cdot05$ p.e. Simple mean $2\cdot03 \pm 0\cdot10$

Note.—The values of Σv^2 are comparable only along horizontal lines as the number of stars and also the decimal places are different for each year.

of A as derived. It is obvious that a solution by the method proposed by Danjon must give a still smaller value of A as in this method the weight of the nearest stars is greatly increased. In fact we have obtained in this case $A = 1''\cdot 12$.

Table I gives a general survey of the results obtained so far.

The best overall value at present available is $A = 2''\cdot 0$ but I would like to stress once more that it has been obtained assuming the inverse distance law of deflection.

In 1924 Professor Esclangon, the director of the Paris Observatory, wrote: "La seule conclusion légitime à tirer est que ces observations sont encore impuissantes à élucider la question posée. Elle ne confirment ni n'infirment la loi de déviation d'Einstein. Elles semblent d'indiquer seulement, si l'on peut écarter vraiment toute hypothèse d'erreurs systématique, l'existence de déviations au voisinage du Soleil sens qu'on puisse en fixer la loi, ni l'exacte grandeur au bord solaire."

I am afraid that now, after thirty-five years and six successful observations of the Einstein effect, the same rather sceptical words can be repeated once again. I think that in order to bring more light into this very difficult problem from an astrometrical point of view, observations more coordinated on an international scale should be made. It seems to me that the method which employs a plane-parallel glass plate before the objective-lens of the astrograph ensures the best independent determination of the difference of scale, thereby increasing the trustworthiness of the result.

There is yet another flawless method of an instrumental determination of the scale correction. Academician V. Linnik of Leningrad has devised an attachment consisting of a bridge, bearing a quartz prism, which is swung into position over the lens of the astrograph and gives on the photographic plate, near its edges, two sets of interferometric fringes divided by a strictly constant angle. By measuring the linear distance between these fringes any change in the scale can be determined. The inconvenience of this method lies in the necessity of switching this attachment into position during each exposure of the eclipse field and then quickly removing it before the next exposure, thereby taking away some of the precious time of the eclipse. If this circumstance is permissible then it could be recommended that both methods be used simultaneously, i.e. taking photographs with the plane-parallel plate in position and also printing-in the standard angle. This would give a perfect check on the determination of the scale as the printing-in of the angle is independent of any refraction anomalies of the outer air which could affect the check field obtained by means of the plane-parallel plate.

One more question: is there any possibility of observing the deflection of light on some other occasion than a solar eclipse? We have seen that the deflection of light is proportional to the potential of attraction. The next massive body in the solar system after the Sun is Jupiter. Its mass is 1047 times smaller than the Sun's and the mean diameter 10 times less. Thus the potential on the surface of Jupiter is 105 times smaller than on the solar surface and the deflection of light is therefore:

$$\Delta r = \frac{0''\cdot 0167}{r},$$

r being the angular distance of a star from the centre of the planet expressed in units of its apparent radius. It is seemingly hopeless to observe such a small deflection directly but there is still one possibility. Suppose that a double star

of some 20″ separation is occulted by Jupiter. There will be a moment when one of the components is nearly grazing the planet's disk, while the other is 20″ away from its limb. Jupiter's apparent radius may also be 20″. In this case the angular distance between the components of the double star will be diminished by half the constant, i.e. 0″·008. This amount can be detected by very precise interferometric measurements and as the duration of the occultation can last several hours there is a possibility of making several measurements with varying r, thus checking the law of deflection. It is obvious that the most advantageous case would be when one of the components passes tangentially to the rim of the planet while the other is situated at right angles to the direction of Jupiter's motion, remaining outside the disk. The probability of such an occurrence is very small but still it is worth-while to be on the lookout for such a chance.

One last remark about a curious phenomenon. Let there be an optical double star, the components of which are at very different distances from the Earth. Because of the relative proper motions of these stars it may occur that one of them passes behind the other. There will be a time when they are nearly aligned. In this case the nearest star will act as a lens and the rays from the farthest star by bending when passing the nearer star will form around it a bright ring of very small angular diameter which could pass unnoticed, except that the star would flash-up for a short time. This rather fantastic occurrence was theoretically investigated by the oldest living Soviet astronomer, Professor Tikhov. So when a nova bursts forth on the heavens there is just a possibility that this is a tribute to Professor Einstein and his prediction of the action of gravitation on light; but I am almost sure that such a phenomenon will never happen.

NOTE ON COLLISIONAL DISSOCIATION OF THE H^- ION IN THE SOLAR ATMOSPHERE

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Summary

New data on collisional dissociation of H^- ions suggest that electron impact is a negligible factor, but that associative electron detachment by impacts with neutral hydrogen atoms is powerful enough to maintain an H^- : H ratio in accordance with local thermodynamic equilibrium in the photospheric layers.

In a study of the continuous emission near the solar limb (Pagel 1956), it was shown that the source function is affected by any departures from ionization equilibrium in the abundance of the absorbing ion. If photo-ionization and electron impact are the only important processes causing dissociation of H^-, the degree of dissociation x is given in terms of the equilibrium degree of dissociation $x^{(T)}$ by

$$\frac{x}{1-x}\Big/\frac{x^{(T)}}{1-x^{(T)}}=\frac{N_eS+Q}{N_eS+Q^{(T)}}, \tag{1}$$

where $N_e S$ is the probability of dissociation of an H^- ion per unit time by electron impact, Q the corresponding probability of photo-ionization, and the superscript $^{(T)}$ refers to local thermodynamic equilibrium. In the chromosphere, Q has the constant value of $1{\cdot}26\times10^6\,\mathrm{sec}^{-1}$ and may differ from the local value of $Q^{(T)}$ by as much as 50 per cent ($Q=Q^{(T)}$ at $T=4800°$, approximately); consequently it is important to investigate the value of S in order to determine to what extent the departures from equilibrium ionization due to the radiation field are damped down by electron impacts.

In the paper already quoted, S was estimated roughly by using J. J. Thomson's classical formula for hydrogen-like atoms and it was concluded that $N_e S$ was comparable with Q at levels of the solar atmosphere having electron density $N_e\leqslant10^{12}\,\mathrm{cm}^{-3}$, and greater than Q at deeper levels. This conclusion now has to be revised in the light of a new computation by Geltman (1959) of the collisional dissociation cross-section of H^- by electron impact, using the Born-Oppenheimer approximation together with a correction for the long-range Coulomb repulsion between the incident electron and the negative ion. Owing to this latter effect, the values of S obtained by Pagel (1956) are not even roughly correct, being too large by a factor of several hundreds. Values of S based on Geltman's cross-sections are given in the following table.

TABLE I

Electron temperature	S	S (Pagel 1956)
4000°	1.15×10^{-9} cm^3 sec^{-1}	
4334°	2.09×10^{-9} cm^3 sec^{-1}	1.1×10^{-6} cm^3 sec^{-1}
5000°	5.56×10^{-9} cm^3 sec^{-1}	
5800°	1.22×10^{-8} cm^3 sec^{-1}	2.7×10^{-6} cm^3 sec^{-1}
6000°	1.46×10^{-8} cm^3 sec^{-1}	

Consequently, electron impact dissociation is negligible compared to photoelectric dissociation for layers of the solar atmosphere having $N_e \leqslant 10^{14}$ or $\tau_{5000} \leqslant 1.5$.

Before drawing any conclusion from this as to the departures from equilibrium in the dissociation of H^- near the limb, it is necessary to consider a further effect, to which attention has been called by Dalgarno, namely *associative detachment* in the reaction

$$H + H^- \rightarrow H_2 + e.$$

Since the rate of this reaction and its converse are dependent on the particle densities and on the kinetic temperature only (assuming equal kinetic temperatures for the electrons and the heavy particles), this process tends to preserve thermodynamic equilibrium provided that equilibrium can be assumed in the abundance of H_2, and equation (1) should be replaced by

$$\frac{x}{1-x} \Big/ \frac{x^{(T)}}{1-x^{(T)}} = \frac{N_H D + N_e S + Q}{N_H D + N_e S + Q^{(T)}}, \tag{2}$$

where $N_H D$ represents the rate of associative detachment per H^- ion. Precise calculations of D are not available, but from general considerations Dalgarno (1958) has estimated that D should be within a factor 4 of $10^{-9.5}$ cm^3 sec^{-1} at the relevant temperatures. Table II gives the values of $N_H D$ for various optical depths based on this estimate and on the model previously adopted.

TABLE II

τ_{5000}	0.00001	0.0001	0.0016	0.003	0.015
$N_H D$	1.5×10^4	1.1×10^5	1.4×10^6	2.5×10^6	8.1×10^6

It is clear from Table II that $N_H D$ becomes greater than Q for $\tau_{5000} > 0.002$, a level which is some 50 km above the solar limb, so that in those regions where there is appreciable self-absorption in the continuum, it appears to be correct to assume the validity of Saha's equation. In the range $0.0002 < \tau_{5000} < 0.001$, it still seems best to assume that collisional and radiative detachment are of comparable frequency, but at the greatest heights ($\tau_{5000} < 0.0002$, height above limb > 500 km), the abundance of H^- in this model would be about 2.5 times that expected from Saha's equation instead of 1.5 times this (the value assumed by the writer in 1956). Since this region is effectively transparent, the values of temperature and electron pressure inferred from the emission *at a given height* remain unaffected by the departures from equilibrium; in this region these were taken from the work of Athay, Menzel, Pecker and Thomas (1955). However, the corresponding *optical depths* less than 0.0002 given in the model should be increased by a factor of about 1.7 in view of this result. However, should the large value of D be confirmed by a detailed investigation, it will be necessary to consider carefully the dissociation equilibrium of H_2.

I am grateful to L. M. Branscomb and R. N. Thomas for some valuable discussion and to S. Geltman and A. Dalgarno for their kindness in communicating their results in advance of publication.

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1959 *April.*

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ON THE DETERMINATION OF EPHEMERIS TIME

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Summary

It is pointed out that there are two inconsistencies in the set of values of ΔT given in the *Astronomical Ephemeris* for 1960: (i) a discontinuity at 1923 due to a change in the adopted equinox, and (ii) a change at 1923 in the value of the Earth's ellipticity used in computing the Moon's tabular place, which gives rise to an erroneous term with the period of the revolution of the node.

Recommendations are made for future discussion of lunar observations and the definitive determination of ephemeris time.

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1. *Introduction.*—The *Astronomical Ephemeris* for 1960 gives smoothed annual values of ΔT from 1900·5. Up to 1948·5 these values have been taken from a paper by Brouwer (1). The purpose of this note is to draw attention to certain inconsistencies in the reductions of the lunar observations which affect Brouwer's values of ΔT, and to make some recommendations concerning the definitive determination of ephemeris time.

Before 1923 (when Brown's *Tables* were first used in the ephemerides) Brouwer's results depended systematically on the two occultation discussions of Spencer Jones (2), (3). The equations of condition for the two series were reduced by Spencer Jones to the system of Brown's theory by the application of the larger terms in the differences between the latter and Hansen's *Tables* on the one hand and Newcomb's "provisionally accepted theory" on the other. The Greenwich and Washington meridian observations for this period had also been reduced approximately to Brown's theory. However, Brouwer corrected these results empirically to the system of the occultations, so that any inconsistencies in the meridian reductions will have very little effect on his values of ΔT.

Since 1923 the occultation results are those of the annual discussions, and, from 1932 onwards, the star positions used have been reduced to the system of the *Zodiacal Catalogue* (4), which is essentially the same as FK3. From 1923 to 1931 Brouwer corrected the occultations by an empirical linear formula obtained from comparison with the meridian observations.

2. *Ellipticity of the Earth.*—Although the occultations before 1923 have been reduced approximately to Brown's theory, the results derived from them differ in one important respect from those which would have been obtained if Brown's *Tables* had been used. The value of the ellipticity of the Earth used by Spencer Jones in both his discussions was 1/297, and the Greenwich and Washington meridian observations before 1923 were reduced with 1/297 and 1/298·2 respectively (5), (6). The value adopted by Brown in constructing his *Tables* was however 1/294. If it is assumed that 1/297 is correct, then, as has been pointed out elsewhere (7), adherence to 1/294 in the ephemeris will introduce an erroneous

term with a period of 18·6 years into B, the fluctuation in the Moon's mean longitude. It is thus desirable that Brouwer's values of B after 1923 should be corrected by $+0''\cdot15 \sin \Omega$ where Ω is the longitude of the Moon's node, in order to bring them into accordance with those before that date.

In any future discussion of lunar observations, corrections should be applied to the ephemeris to reduce the tabular places to the best available value of the ellipticity. It would certainly be better to use the international value 1/297 rather than 1/294.

If e is the true ellipticity, then an error in frequency measured in terms of the rate of change of ΔT, obtained from comparison of observations of the Moon with the lunar ephemeris, is approximately $10^{-9}(e^{-1}=294) \cos \Omega$. Recent observations of artificial satellites indicate that e may be near 1/298 (8); the corresponding error in frequency will thus be $4\times10^{-9} \cos \Omega$.

3. *Equinox error.*—Brouwer states (1, p. 128) "... Newcomb's equinox ... is the equinox used in Spencer Jones' revision." This statement appears to be erroneous. In the revision of Newcomb's occultations (3) a correction to the assumed right ascensions of the stars in Newcomb's fundamental catalogue was included in the equations of condition. We infer, therefore, that Spencer Jones' derived values of the mean longitude are independent of an error of equinox. The equinox correction which he derived for epoch 1850 was $-0^{s}\cdot047$; there is considerable uncertainty as to a possible mean motion, but we may take $-0^{s}\cdot05$ as the correction required by Newcomb's right ascensions at epoch 1900. The correction to the observed mean longitude corresponding to an equinox correction E (seconds of time) is $15E \cos \epsilon$ where ϵ is the obliquity of the ecliptic. Thus for $E=-0^{s}\cdot05$ the correction to the observed mean longitude is $-0''\cdot69$.

The Cape occultations discussed by Spencer Jones (2) are referred specifically to Newcomb's right ascension system, and Brouwer found that an empirical correction of $-0''\cdot64$ was required to reduce them to the system of the revision of Newcomb's occultations. This is almost identical with the expected equinox correction. We may conclude, therefore, that before 1923 Brouwer's values of B are all independent of an error of equinox.

Since 1923 the observations used by Brouwer are referred to adopted right ascensions which may be expected to be largely free from an error of equinox. However, Brouwer, supposing that his values of B before 1923 were all referred to Newcomb's system, introduced a discontinuity of $+0''\cdot6$, which is equivalent to $+1^{s}\cdot09$, into ΔT. All his values of ΔT since 1923 should thus be decreased by this amount.

4. *Definitive determination of* ΔT.—The comprehensive discussion of Brouwer, as amended by the removal of the inconsistencies referred to above, contains the most homogeneous set of values of ΔT which are at present available.

Since 1923 the occultations have been reduced by lunations, and corrections obtained to the orbital longitude and latitude only. As has been remarked elsewhere (7), the long series from 1923 to 1959, extending over the whole period during which Brown's *Tables* have been in use, covers almost exactly two complete revolutions of the node. A discussion of these, and the meridian observations for the same period, should be made in order to obtain definitive values of ΔT as well as corrections to the orbital elements of the Moon. Such a discussion should include a possible equinox correction and mean motion. The importance of using the correct equinox, as distinct from one arbitrarily defined by a system of

adopted right ascensions, arises from the fact that an equinox correction E results in a correction of approximately $25E$ to ΔT, and a mean motion of this will affect the rate of change of ΔT. It has been shown elsewhere (7) that the occultations give an equinox correction to FK3 which is in good agreement with that obtained from meridian observations of the Sun and planets.

Consideration should also be given to a rediscussion of all the lunar observations back to at least 1850 using an ephemeris rigorously computed from Brown's theory. Brouwer points out that the agreement between the meridian and occultation results has been very much better since 1923 than it was before that date, and attributes this partly to imperfect differential correction of the meridian observations from Hansen's *Tables* to Brown's theory. The marked increase in the scatter of the annual means from occultations before that date indicates that these too could be improved by such a rediscussion.

5. *Ephemeris Time.*—A practical system of uniform time is defined uniquely by two arbitrary constants, an epoch, and a unit of time. The epoch 1900 January $0^d\ 12^h$ Ephemeris Time (E.T.) is defined to be (9) "...the instant, near the beginning of the calendar year A.D. 1900 when the geometric mean longitude of the Sun was $279°\ 41'\ 48''\cdot04$...", and the unit of time is the ephemeris second as defined by the Comité International des Poids et Mesures (10), which is the time taken for the Sun's observed mean longitude to increase by $\frac{129\,602\,768''\cdot13}{3\,155\,760\,000}$ at epoch 1900 January $0^d\ 12^h$ E.T.; the numerator in this expression is the mean motion of the Sun in a Julian century as given in Newcomb's *Tables*, and the denominator is the number of seconds in a Julian century. The need for specification of the epoch in the definition of the unit of time arises because the actual motion of the Sun is accelerated.

The system of E.T. is thus defined uniquely in terms of two of the arbitrary constants of the Earth's orbit. The numerical values have been chosen to be the same as those adopted by Newcomb in constructing his *Tables*. If at some future date an ephemeris of the Sun with argument E.T. is to be constructed using an improved set of tables of the motion of the Earth, then the numerical values of these two arbitrary constants must be the same as those assigned by Newcomb.

Because E.T. is defined in terms of the mean longitude of the Earth, its practical determination must depend ultimately on observations of the Sun. But on account of the relatively small mean motion of the Sun, the I.A.U. has recommended the use of lunar observations and defined the quantity ΔT in terms of B (11). The two time systems E.T., and U.T. + ΔT are however not logically the same. The determination of ΔT from the lunar ephemeris depends on the constants of the Moon's mean longitude used in that ephemeris, so that before ΔT can be used as an approximation to E.T.–U.T. these arbitrary constants must be determined from observations. A further complication in using lunar observations is the existence of an empirical secular retardation term in the expression of the Moon's tabular mean longitude (12); this has been emphasized by Atkinson (13).

In his fundamental work on the rotation of the Earth (14), Spencer Jones related the mean longitudes of the Sun, Mercury and Venus to that of the Moon. This work is based on observations extending from the latter half of the 17th century to 1936. For the greater part of this period, values of B were taken from his discussion of Newcomb's occultations (3). However, from 1908 his values depend partly on Greenwich meridian observations and partly on the annual

occultation discussions, and from this date onwards there is a systematic difference of nearly 1″ between the values he used and Brouwer's values amended for the error of equinox referred to above. A new discussion of the modern observations of the Sun and planets using definitive values of ΔT from the rediscussion of the lunar observations would improve the determination of ephemeris time over the last hundred years. An important consequence of such a discussion would be a new determination of the empirical secular term in the Moon's mean longitude.

6. *Conclusions.*—The points raised in this note may be summarized in the following specific recommendations :—

(i) A comprehensive discussion of all lunar observations from 1923–1959 should be made in order to derive definitive values of ΔT, as defined in (11), and corrections to the elements of the Moon's orbit.

(ii) All lunar observations from 1850 to 1922 should be rediscussed using an ephemeris of the Moon computed from Brown's theory.

(iii) Each of the above discussions should be based on the best available value of the Earth's ellipticity.

(iv) The derived values of the fluctuations in the Moon's mean longitude should be freed from errors of equinox.

(v) The Sun and planet observations from 1850 should be rediscussed using the values of ΔT obtained in (i) and (ii) above to give a definitive determination of ephemeris time, as defined in (9).

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1959 *May* 12.

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THE FABRY–PEROT MONOCHROMATOR

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Summary

Many astronomical spectrographs are inefficient in the sense that light from the telescope image is both rejected at the entrance slit and wasted by the low quantum efficiency of the photographic plate. There are many problems for which the second of these losses may be reduced by the use of a photoelectric detector. It is shown that in such cases the addition of a Fabry–Perot interferometer to a conventional monochromator will allow the use of a much wider entrance slit for a given resolution. The theory of this Fabry–Perot monochromator is considered and leads to the conclusion that the interferometer is best used in an on-axis arrangement. Such a system has been developed and tested experimentally both in the laboratory and with a 120 cm telescope. Comparisons have been made between a prism monochromator used to study short regions of the spectrum of stars and nebulae, both alone and with an interferometer in an on-axis and an off-axis arrangement. The results show that the on-axis arrangement gave a gain of light of about thirteen, and was limited by the goodness of the surfaces of the optical flats used. The system described can be attached to any existing monochromator.

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1. *Introduction.*—It is well known that the high quantum efficiency of the photomultiplier may be usefully employed in astronomical spectrophotometry for problems involving a small wavelength range (1). The associated spectrographic equipment can then be a monochromator which scans a series of spectral elements sequentially. Jacquinot (2) has shown that under these circumstances a Fabry–Perot interferometer (F–P) is greatly superior to a grating or prism of the same size. This is because the F–P permits the use of a much wider slit for a given resolution. The application of the F–P to experimental astronomy has been discussed theoretically (3) and attempted practically (4), but with a wide divergence between theory and attainment. The theory was based on unrealistic estimates of the quality of optical surfaces and the experiments were made with a simple scanning system which did not use an F–P to best advantage. It is the purpose of this communication to discuss the practical limitations of the method and to describe some experiments carried out with an F–P used to better advantage.

2. *Theory of the diffraction-grating monochromator.*—Jacquinot has shown that a grating achieves its optimum efficiency when working in the Littrow arrangement. Under these circumstances the width of the entrance slit subtends an angle ϕ at the collimator such that

$$\phi = \frac{2\tan\theta}{R} \tag{1}$$

where θ is the blaze angle of the grating and R is the geometrical resolving power of the spectrograph, which is assumed to be smaller than its theoretical resolving power. When this spectrograph is combined with a telescope so that the focal

ratio of the telescope is the same as that of the spectrograph collimator, the slit width projected on the sky subtends an angle ϕ', where

$$\phi' = \frac{2 \tan\theta \,.\, D_g}{R \quad D_t}, \tag{2}$$

D_g is the diameter of the spectrograph collimator, and D_t is the diameter of the telescope objective. Fig. 1 shows this relationship for a diffraction grating with a blaze angle of 30 °.

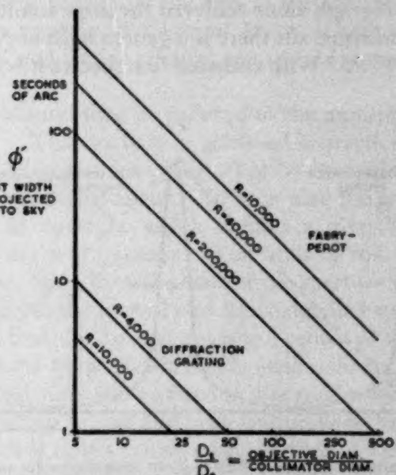


FIG. 1.—*Comparison of the angular slit widths for Fabry–Perot and grating monochromators used with a telescope at various wavelength resolutions.*

In practice the gratings used are rarely large enough for the slit to admit all the light from a scintillating star image, except at low resolution. For example (5) the coudé spectrograph of the 100 inch telescope at Mt Wilson, which has an 8 inch grating, begins to lose an appreciable amount of light at the slit when the resolving power is about 4 000. Problems exist which require resolving powers of at least 200 000, and the use of an F–P in series with the grating permits the resolving power to be increased without the slit being narrowed.

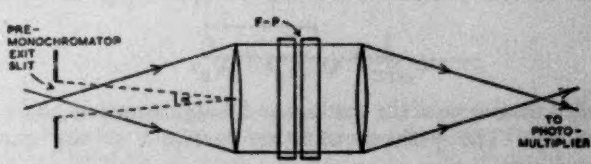


FIG. 2.—*Optical system of a Fabry–Perot monochromator.*

3. *Theory of the Fabry–Perot monochromator.*—The Fabry–Perot interferometer placed in a collimated beam (Fig. 2) acts as an interference filter with a number of transmission bands whose width and spacing can, within limits, be suitably chosen, and whose position in the spectrum can be continuously varied.

If one of these bands is isolated by an auxiliary tunable filter (a pre-monochromator), the arrangement becomes a scanning monochromator. An existing spectrograph may conveniently be used as this auxiliary filter and need only have an instrumental profile whose base is twice as wide as the separation of the transmission bands (W). The resolving power of the complete system is then that of the interferometer (Fig. 3).

If ω is the width at half intensity of one of the bands, the ratio W/ω is called the finesse (N). The spectrograph slit may thus be N times wider than would be the case if the spectrograph alone achieved the same resolution. For images that are larger than the widened slit there is a gain in light of NT, where T is the peak transmission of the F–P. With emission line spectra it is often possible to exceed

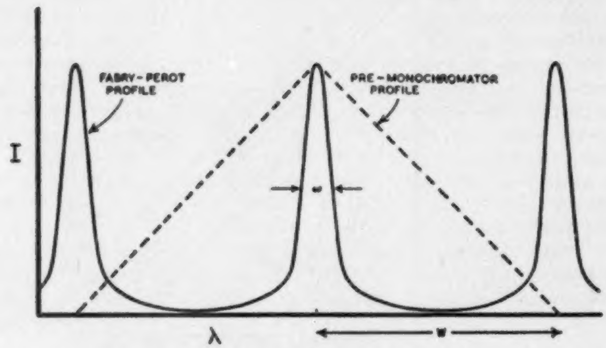


FIG. 3.—*Intensity profiles for a Fabry–Perot monochromator and its associated pre-monochromator.*

this gain, since the spectrograph slit may be further widened, thus allowing several of the transmission bands to pass (provided that light is not admitted in these additional bands which will interfere with the observations).

The maximum value which the product NT may take for a practical F–P has been discussed by Chabbal (**6**), and his conclusions may be summarized as follows. Any pair of optical flats will have surface irregularities which ultimately limit the attainable finesse. If N_D is this limiting finesse, it is because the amplitude of the surface defects is about $1/N_D$ of a fringe. To attain this finesse the surfaces must be given a very high reflectivity, and if the reflectivity is lowered the effective finesse will decrease to a value N_E given approximately by

$$\frac{1}{N_E} = \sqrt{\frac{1}{N_D^2} + \frac{1}{N_R^2}} \tag{3}$$

N_R is called the finesse of the coatings and is approximately $3/(1-R)$, R being their reflectivity. The peak transmission increases as the coating finesse decreases, such that

$$T = N_E/N_R, \tag{4}$$

neglecting absorption losses.

The reflectivity of the coatings should be chosen to maximize $N_E T$, and from equations (3) and (4) the maximum value of this product occurs when $N_R = N_D$ so that

$$N_E T = N_D/2. \tag{5}$$

Although the reflectivity of the coatings should be matched to the surface quality of the flats used, this matching is not critical. $N_R T$ will exceed 80 per cent of its maximum value if

$$N_D/2 < N_R < 2N_D. \tag{6}$$

The above analysis shows that the gain in light obtained for a given resolution is directly proportional to N_D, i.e. the flatness of the interferometer plates. It is found in practice that interferometer plates possess large-scale irregularities, which means that N_D will depend on the area of surface used. We have made measurements of N_D on a good pair of optical flats with high reflectivity coatings, and found values of 28 and 31 for circular areas of 10 mm and 3 mm diameter respectively.

The interferometer should thus be reduced to the minimum permissible size in order to increase N_D. This can only be achieved in practice by either reducing the focal length of the collimator lens (Fig. 2) or by increasing the focal ratio of the beam from the pre-monochromator. In each case the angle (α), subtended at the collimator by an aperture which defines a given range in the pre-monochromator spectrum, will increase. The limit to this angle is set by the resolution required (R). Rays from the edge of the aperture will strike the interferometer at a small angle to the normal and the channelled spectrum of this light will be displaced from that due to rays from the centre of the aperture. This effect broadens each of the transmission bands (also reducing its peak intensity) and the permissible broadening will be fixed by the resolution. A graph is given (Fig. 4) of the way in which finesse and peak transmission vary with $R\alpha^2$. The entrance aperture, which is now a circular hole instead of a slit, should be made to subtend as small an angle as possible by using a relatively large area of optical flat, but for any pair of flats there will be a compromise between the size set by this requirement and that set by the irregularities of the surfaces.

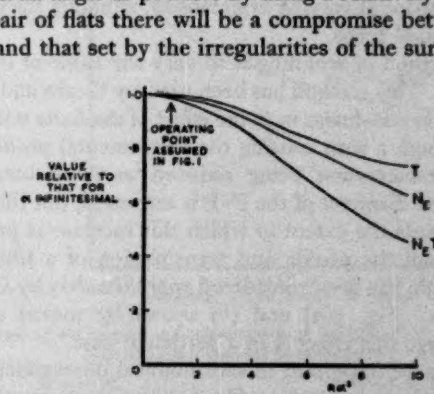


FIG. 4.—*Variation of transmission T, finesse N_E and their product $N_E T$ with $R\alpha^2$ (R=resolution, α=angular slit width in radians) for an on-axis F-P. The curves are plotted from data given by Chabbal* (7).

It is possible to make the effect of the finite size of the aperture very small if we fulfil the condition

$$\alpha^2 = 1/R \tag{7}$$

and even for the highest resolution at the focus of the largest telescope the flats required for ϕ', say 5 seconds of arc, are still very small (Fig. 1), so that the effects of finite aperture need reduce our previously deduced gain by only a few per cent.

4. *Wavelength scanning.*—There are three ways of varying the wavelength passed by an F–P. The wavelengths of the pass bands, λ, are given by

$$\lambda = \frac{2\mu t \cos\beta}{p} \tag{8}$$

where t is the separation of the plates, and p (the order of interference) is an integer: μ is the refractive index of the medium between the plates and β is the angle at which the light is refracted in the spacer medium. It is thus possible to vary λ continuously by variation of either μ, t, or β.

There is often a practical limit to the range of wavelength which may be covered by varying one parameter alone. It is possible with some methods to increase this range by "sawtooth scanning" in which the F–P and pre-monochromator are scanned together through an exact number of orders of the interferometer. The pre-monochromator then remains at a fixed wavelength whilst the F–P is quickly returned to its original state, whereupon the scan may continue. The saw-tooth method is only necessary when the range of wavelengths to be scanned is large. When this range is many (say 100) times the instrumental resolution the advantage of speed which the photomultiplier enjoys over the photographic plate is lost. There remain the advantages of linearity, lack of saturation and immediate presentation of results.

Mechanical means of varying the plate separation have been tried in several laboratories, and systems using springs, piezo-electric effect and thermal expansion have been developed (**8**). The F–P plates must be maintained so nearly parallel that any change in separation across the plates during a scan is much smaller than λ/N_D. This would be difficult to achieve for a mechanically scanned interferometer used on a telescope at varying orientations, although the method might well be employed at a coudé focus.

The simplest method of scanning is to vary the angle of incidence by tilting the interferometer. This method has been used by Geake and Wilcock (**4**). As the wavelength scan is non-linear in β, the effect of the finite width of the entrance aperture varies through a scan causing the instrumental profile to change, both finesse and peak transmission being reduced as β increases. These effects become smaller as the diameter of the F–P is increased, but the quality of the flat surfaces available limits the extent to which this increase is profitable.

The way in which the profile and transmission of a tilting interferometer vary throughout a scan has been considered approximately by Geake and Wilcock and by the authors. Fig. 5(*a*) and (*b*) shows by means of graphs derived theoretically how large this effect is in a particular case.

These effects can be important in astronomical observations. For example, this variation of the instrumental profile during a scan greatly complicates the study of line profiles, while the change in transmission involves a large correction when the relative intensities of lines are to be determined. In all scanning systems there are other complications due to the variation of illumination of the entrance aperture caused by "seeing". These complications may be avoided for the on-axis F–P if its size fulfils the conditions we have given (eqn. (7)), but they are more important for the tilting F–P. An analysis of these effects will be published in detail elsewhere.

The method of scanning most suitable for use on a telescope is that of variation of the refractive index of the spacer medium by change of gas pressure. It is

easily shown that a variation $\delta\mu$ in spacer refractive index will change the wavelength of a pass-band originally at λ by an amount $\delta\lambda$ given by

$$\frac{\delta\lambda}{\lambda} = \frac{\delta\mu}{\mu}. \tag{9}$$

For small changes in the pressure of a gas, the refractivity $(\mu - 1)$ is proportional to pressure, and as the refractive index of air is 1·0003 at NTP, a change of one

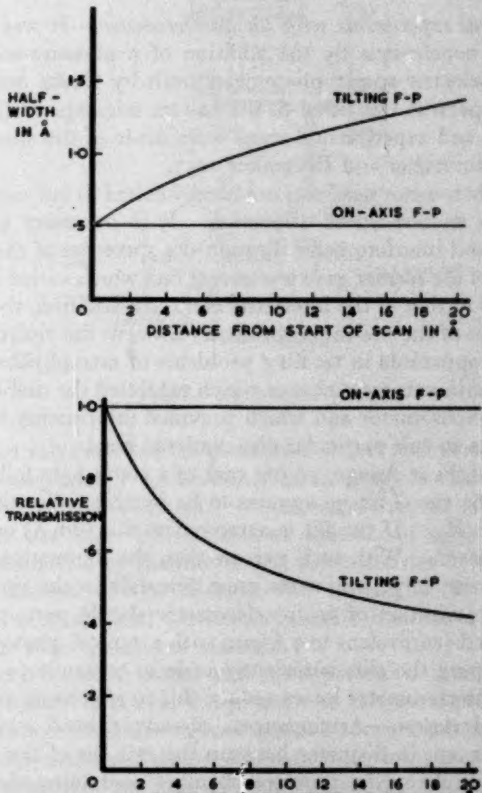


FIG. 5.—*Deterioration of the profile and transmission of a tilting F-P compared to an on-axis F-P as each is scanned in wavelength.*

W = 10 *A*. *w* = 0.5 *A*.
α = 15 *min. of arc.* λ = 4340 *A*.

atmosphere in the pressure of an air spacing layer will move the interferometer pass-bands 1·2A at λ4000. Previous workers (9) have used such pressure changes, but when using the interferometer astronomically it is often necessary to scan a range greater than 1A. It was therefore desirable to find gases of higher refractivity than air and also to use pressures of several atmospheres. This latter requirement was found to simplify the problem since pressure controllers to deal with a few atmospheres pressure are readily available, whilst experimenters working with lower pressures have normally developed their own pressure controls.

Table I lists some of the gases that have been found useful for our purposes, together with the vapour pressure at 0 °C and the range scanned per atmosphere at 5,000 A.

TABLE I

Gas	Oxygen	Arcton 12 ($CClF_2$)	Propane
Range (A/atmos.)	1·5	3·5	5·5
Vapour pressure at 0 °C (atmos.)	—	34	6

5. *Astronomical experiments with an interferometer.*—It was decided to test these theoretical conclusions by the addition of a pressure-scanned interferometer to a photoelectric spectrophotometer built by Geake and Wilcock (**10**). The complete apparatus was fitted to the 120 cm telescope of the University of Padua at Asiago, and experimental scans were made of the spectra of stars and nebulae during November and December 1957.

The spectrophotometer used was not ideally suited to our needs. One of the difficulties lay in its non-linear dispersion. It is necessary to scan the pre-monochromator and interferometer through the spectrum at the same rate, and the prism optics of the former gave a scanning rate which varied with wavelength in a different way to that of the interferometer. In addition, the rapid decrease of the transmission of the pre-monochromator towards the violet severely limited the utility of the apparatus in tackling problems of astrophysical interest. For these reasons experiments were chosen which exhibited the useful features of the Fabry–Perot monochromator and which provided information for the design of future instruments to suit particular observational needs.

On a typical night at Asiago, 70 per cent of a star's light falls inside a slit 4″ wide (although the visual image appears to be much smaller than this) corresponding to 6 A at Hγ. If the slit is narrowed to 0″·4 (0·6 A) only 8 per cent of the starlight is passed. With such narrow slits, the fluctuations in intensity at the photocell are very large, and make great demands on the system of compensation (**10**). The addition of an interferometer should permit a resolution of 0·3 A to be attained (equivalent to 9 A/mm with a typical photographic spectrograph) while keeping the slits sufficiently wide to transmit 70 per cent of the starlight. The interferometer losses reduce this to effectively 50 per cent.

6. *The optical system.*—Arrangements already existed which produced a collimated beam 1 cm in diameter between the exit slit of the monochromator and the photomultiplier; the pressure chamber containing the interferometer was inserted there. The chamber was connected by a flexible tube to a motor-driven pressure controller which was situated on the Newtonian platform together with the gas cylinders, while the monochromator was mounted at the Newtonian focus. The total weight added to the monochromator was 8 lb. The interferometer was of conventional design and contained a pair of 7 cm diameter flats (of which the central centimetre was used) separated by three mica discs about 3 mm in diameter. The flats were adjusted for parallelism using a monochromatic source before commencing observations, and were found to retain their adjustment throughout the night.

7. *Pressure control.*—A pressure system (Fig. 6) was constructed by means of which the pressure around an F–P could be varied over a range of nearly 5 atmospheres. The pressure chamber had ½ inch thick windows of 2 inch clear diameter and was tested to withstand a pressure differential of 120 lb per sq. in. The gas was

led by a flexible metal pipe from a commercial pressure controller* which could be driven through change gears by a synchronous motor. The pressure could be increased linearly with time to a value of up to 65 lb per sq. in. above atmospheric pressure, the maximum value being adjustable. When this value was attained

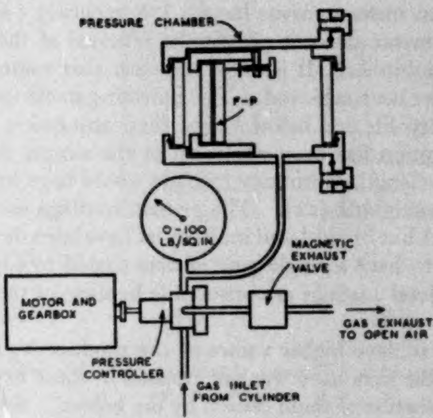


FIG. 6.—*The pressure system.*

a magnetic release valve allowed the pressure to fall to the atmospheric value, and the cycle could be repeated. A Bourdon pressure gauge and an electrical pressure gauge were connected to the system. It was found that the pressure (and hence wavelength) scan was linear with time to an accuracy of 1 per cent but that this accuracy was determined by the mechanical drive and not by the controller. A series of measurements of the linearity and reproducibility of the pressure controller are in progress, and preliminary results indicate a reproducibility of pressure of better than $\pm$ 1 mm of mercury.

No difficulties were experienced with the apparatus in practice. It was however thought advisable to release the propane to the open air after use, though there was little danger of achieving an explosive concentration. Both the prism of the pre-monochromator and the pressure controller were driven by synchronous motors. To synchronize the two scanning rates, the former was driven by a variable frequency oscillator, which was adjusted for each different wavelength region studied.

8. *Finesse and transmission measurements.*—Measurements of the finesse and transmission of the F–P were made by using the helium emission line as a monochromatic source. The light from a helium lamp fell on the entrance slit of the prism monochromator, and the exit slit transmitted the light of a single line. This light then passed through the F–P (Fig. 2) to the detector. As the pressure around the interferometer was varied the instrumental profile was recorded, thus permitting measurements of finesse. The F–P was then removed, and the intensity of the incident light measured. The peak transmission could thus be determined (Table II). This experiment was repeated at several different wavelengths.

* Edwards Type No. VPC1.

TABLE II

Wavelength (A)	3889	4026	4120	4387	4471	4921	5015	5047
Finesse N_E	7	11	10	18	19	15	14	16
Transmission (per cent)	65	65	55	55	50	65	95	85
NT_E	4·5	7·0	5·5	10·0	9·5	10·0	13·5	13·5

The transmission measurements have a low accuracy (± 5 per cent) as the mechanical arrangements did not permit the removal of the F–P without disturbing the photomultiplier. It is, however, clear that values of $N_E T$ (i.e. light gain) of about 13 have been achieved. The reflecting stacks used were made from zinc sulphide and cryolite and below λ4400 their absorption became important. The effect of absorption has been neglected in the simple theory given earlier. For the shorter wavelengths antimony trioxide would have been a more suitable substance than zinc sulphide (**11**). The present coatings are usable only over a range of about 1000A but broad-band multilayers have been developed (**12**), and it should be possible to have a single pair of flats coated to cover the range from λ3500 to 6000. Metal coatings are unsuitable because of their large absorption losses (**13**).

It is possible to achieve higher values of the product $N_E T$ than we attained. The full finesse of the flats used was not realised in these experiments, as there was some minor distortion of them caused by the holder. Several other workers have claimed that their best flats had a limiting finesse of about 50, so for these a value of $N_E T$ of about 25 should be possible.

9. *Tests of the apparatus.*—The apparatus was used on several nights to study the spectra of nebulae and stars, including the planetary nebula NGC 7662, the Orion Nebula, α Aur, ω U Ma and β Per. The discussion will be limited to those spectra for which comparison scans exist showing the performance of the monochromator alone. Two comparisons are given for both emission line objects and absorption line objects, the first being a laboratory test, and the second a test of the apparatus on the telescope.

(*a*) *Emission line objects*

It is often desirable to observe the relative intensities of close doublets in the spectra of nebulae, for which purpose a resolution of 1A is sufficient. A typical case is the (O II) doublet at λ3726, 3729 whose relative intensities lead to a measure of electron temperature and density (**14**). This resolution also allows the radial velocity of nebulae to be determined with a precision of a few km/sec. A laboratory observation of a line showing suitable structure was made on the potassium doublet at λ4044, 4047. The spectrograph slit was weakly illuminated by a rubidium lamp in which potassium was an impurity. The lines were first scanned by the monochromator alone with 50μ slits giving a resolution of 1A. The trace (Fig. 7 (*a*)) is very noisy, owing to the small quantity of light reaching the detector. To obtain a less noisy spectrum in the same observation time, the slits were then opened to 460μ; under these conditions the presence of the lines was established (Fig. 7 (*b*)) but they were not resolved. The F–P was then inserted and the line once more scanned with the wide slit at the same speed (Fig. 7 (*c*)). The resolution was then similar to Fig. 7 (*a*) but the signal/noise ratio had increased.

The low ultra-violet transmission of our pre-monochromator prevented an observation of the λ3726 doublet and so the (O III) lines at λ4959 and λ5007, which exhibit a similar intensity to the (O II) doublet were recorded using several nebulae as sources.

The spectrograph alone was first used on the Orion nebula with 750μ slits, corresponding to a resolution of 32A (Fig. 8 (*a*)). The F–P was then inserted, and the two lines scanned again in the same total time as (*a*), but with 1A

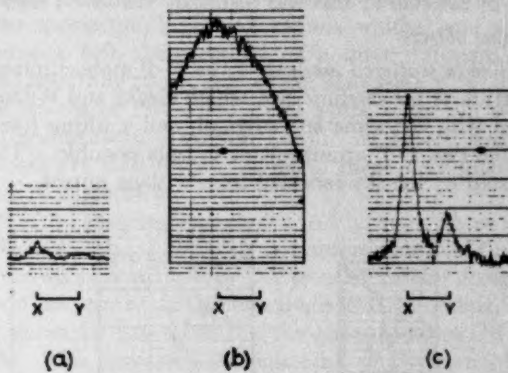


FIG. 7.—*A recording of the potassium lines at* λ 4044 (*X*) *and* λ 4047 (*Y*) *from a discharge tube.*

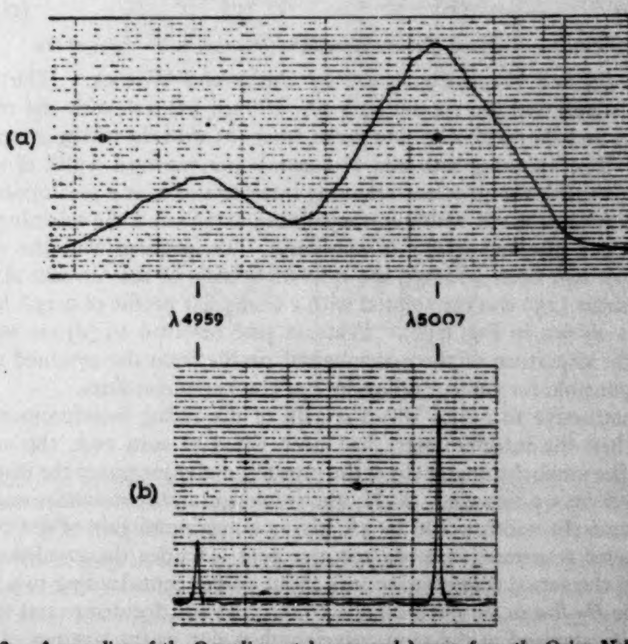


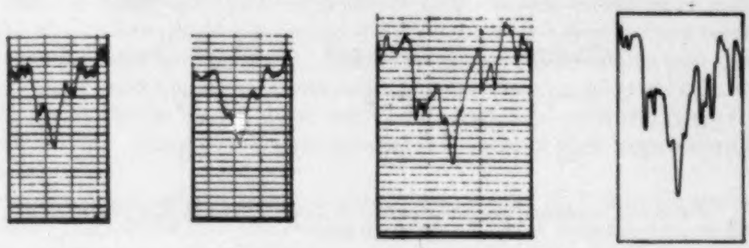
FIG. 8.—*A recording of the O III lines at* λ 4959 *and* λ 5007 *from the Orion Nebula.* (*a*) *Pre-monochromator alone.* (*b*) *With F–P.*

resolution (*b*). Since the approximate separation of the lines was known, the region between, being irrelevant, was omitted. Here the two observations are equally valuable for determining the relative intensities of the lines because of their wide separation but, as Fig. 7 has shown, the F–P technique is necessary for the study

of intensities of close doublets in the spectrum of faint extended objects. For radial velocity determination the F–P scans will always be superior. Although the scans of the Orion nebula with the F–P show no line broadening due to internal motion of the source, this was distinctly visible on scans of NGC 7662.

(*b*) *Absorption line objects*

(i) *The Hγ line in scattered sunlight.*—The F–P monochromator was used to record the spectrum of the daytime sky, which Geake and Wilcock (4) had previously observed with the same spectrograph and a tilting interferometer. A comparison of the two F–P arrangements is thus possible. The spectrograph slit was illuminated by the sky reflected from a plane mirror.



(a) (b) (c) (d)

FIG. 9.—*Spectrum of scattered sunlight (blue sky) in the region of Hγ.*

Traces (*a*) and (*b*) of Fig. 9 are by Geake and Wilcock. The length of spectrum is 20A and the resolution 1·2A. Trace (*a*) was with the monochromator alone and shows photon shot noise, trace (*b*) with the tilting interferometer does not. The scanning rate was 16 seconds per resolved width of spectrum. Trace (*c*) was with the pressure-scanning interferometer at a resolution of 0·25A (pre-monochromator slit width corresponding to 3A) and the scanning rate was 6 seconds per resolved width of spectrum. To confirm that the resolution claimed here had been achieved the relevant section of the Utrecht *Atlas of the Solar Spectrum* (15) was convoluted with a triangular profile of 0·25A half-width and this is shown in Fig. 9 (*d*). Features just resolved in (*d*) are seen in (*c*) although the departure of the instrumental profile from the assumed triangular one is responsible for some "filling-in" of the narrowest lines.

It is instructive to notice why the gain of the tilting interferometer was so small. When the interferometer had been tilted to scan 20A, the angle subtended by the monochromator slit at the interferometer increased the instrumental half-width from 1·0 to 1·2A. At Hγ the resolution of the monochromator alone with the same slit-width would be 5A, giving a maximum gain of 4·2 even if the interferometer transmission were 100 per cent. Under the conditions of the experiment the actual transmission was about 70 per cent, leading to a gain of 3.

(ii) *The Hγ line in the spectrum of o U Ma.* It was decided to test the instrument on a G star and in the same wavelength region as the tracings of the solar spectrum to offer further comparisons. The star chosen was o U Ma ($3^{m}\cdot5$). In a photoelectric spectrograph, scintillation of the star image on the spectrograph slit may be an important source of noise. Normally a compensation system is fitted to the spectrograph to deal with this problem, but for the arrangement used at Asiago the compensation system required a delicate adjustment of the positions of two photomultipliers. The arrangements for inserting the

interferometer prevented this adjustment and this resulted in the tracings with the interferometer being made without the full benefit of the compensation system. This necessitated spectrograph slits wide enough to include most of the star image all the time. Measurements were made to ascertain the fraction of starlight passing the spectrograph slit for various widths, and to determine the intensity fluctuations with these widths. On three different nights, the projected slit widths admitting 70 per cent of the light were $3''$, $3''\cdot 6$ and $4''\cdot 6$, corresponding to slit widths of 170μ, 200μ and 250μ. The slit width adopted to give adequate freedom from noise was 400μ. As before, published scans by Geake and Wilcock are used for comparison; these were made with the compensation system in adjustment.

The tracings of the spectrum are shown in Fig. 10. Those of Fig. 10 (*a*) are with the spectrograph alone. Here the resolution is 2A and the slit width 75μ. Two scans are shown so that features due to noise can be distinguished and on these tracings one can only be certain of a single absorption line. Trace (*b*) with the tilting F–P shows far less noise, but the 400μ entrance slit used limited its resolution to 2A. This prevented the separation of $H\gamma$ from a blend of lines to

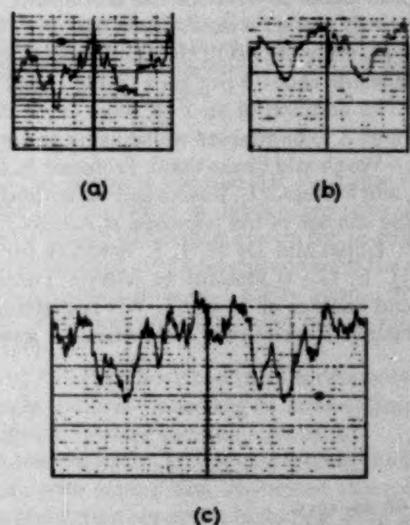


FIG. 10.—*The spectrum of o U Ma covering the same spectral range as Fig. 9. (a) Two successive scans with the monochromator alone. (b) Two scans with the tilting F–P. (c) Two scans with the pressure-scanned F–P.*

the violet, which may be seen by comparison with Fig. 9. The tracings with the pressure scanned F–P (*c*) were made at a resolution of 0·7A (with a premonochromator slit width also of 400μ) and are clearly better, as they show all the features of Fig. 9(*b*). Despite the higher resolution of the pressure scans, all were at the same rate of 20 sec per resolved spectral width. This necessarily introduced photon shot noise in the high resolution tracings.

10. *Conclusions.*—It has been shown that the F–P monochromator can provide a given spectral resolution with wider slits than dispersive instruments alone. The imperfections of optical surfaces limited the gain of light to a value of 13 in our experiments, but it should be possible to extend this by a further factor of

two by using better optical flats. The technique of tuning an F–P monochromator by pressure variation has been extended to allow a range of about 20A, thus permitting the instrument to be used at low resolution.

Astronomical experiments have shown that the F–P technique can be applied to the study of line intensities in emission nebulae. In this field useful observations can be obtained with telescopes of all sizes. Work is in progress on an instrument to study the relative intensities and Doppler shifts in the (O II) doublet (λ 3727) in nebular spectra.

The experiments have also shown that the F–P technique can improve the performance of a photoelectric stellar spectrograph in two ways. The wider slits that can be used reduce the intensity fluctuations caused by movement of the stellar image over the slit and at the same time allow an increased amount of light to fall on the detector. A small telescope may be equipped with a diffraction grating large enough to allow all the light in a stellar image to pass when used at low or moderate resolution, and therefore does not need an F–P. The low flux-gathering power of such a telescope will handicap its use for observations at high resolution. However, large telescopes cannot at present be fitted with gratings that are large enough to allow an efficient use of the flux collected from stars except at low resolution. It is to such instruments that an F–P stage may be added advantageously. For example the slit of the coudé spectrograph attached to the 100 inch telescope transmits about 6 per cent of the light from a star when the resolution is 0·1 A. The addition of an F–P stage (1 cm diameter flats) would permit a resolution of 0·05 A to be attained whilst using 50 per cent of the starlight.

Acknowledgments.—We should like to thank Professor S. Devons for providing facilities for our work, and Professor L. Rosino and the authorities of the University of Padua for permitting the use of the telescope at Asiago. We should also like to thank Professor Z. Kopal and Dr H. J. J. Braddick for encouragement and advice. One of us (J. E. G.) is grateful to Messrs Turner and Newall for a research fellowship, and another of us (N. J. W.) is grateful to the Department of Scientific and Industrial Research for a maintenance award.

The University,
Manchester, 13:
1959 *January*.

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THE EMISSION LINE SPECTRUM OF THE ECLIPSING VARIABLE AR PAVONIS

A. D. Thackeray

(Received 1959 May 14)

Summary

The bright line spectrum of AR Pav is described. During eclipse the intensity of the H lines weakens relative to the nebular lines; He I weakens more, while He II 4686 nearly disappears. Stratification as in a planetary nebula is suggested.

In the P Cyg structure of the Balmer lines the red component is always stronger than the violet, but equality is approached for over 100 days following eclipse. The separation of the components narrows during eclipse by an amount which varies in different cycles. Outside eclipse the mean separation obeys Struve's relation $V^2 \propto P^{-1}$ for gaseous rings.

A model of a thick ring subject to differential rotation is considered and profiles computed geometrically for various types of eclipse. The observed narrowing of components is not reproduced in this model. As an alternative an expanding ring is considered briefly, and this model predicts symmetrical narrowing during eclipse.

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1. *Introduction.*—The eclipsing variable AR Pav* discovered by Mrs Mayall (1), who discussed the light-curve, is of outstanding interest for several reasons. The period (605 days) is among the longest known for an eclipsing variable. The light-curve shows fluctuations of over a magnitude from the mean curve.

Added to this the hydrogen lines of AR Pav appear bright with P Cyg-like structure. The only other known P Cyg star which varies regularly like an eclipsing variable is GG Car.

Radcliffe spectra of AR Pav, described in an earlier note (2), showed that the spectrum is exceedingly complex, consisting of three or four parts:

(1*a*) a high excitation nebular emission spectrum;
(1*b*) a relatively low-excitation emission spectrum (Fe II and [Fe II]);
(2) a continuum with superposed supergiant absorption (Ti II etc.); and
(3) a late-type absorption spectrum including Ti O bands.

(1*a*) persists through eclipse, (1*b*) is more easily seen during eclipse, (3) is seen only during eclipse. (2) was peculiarly strong during emergence from the 1954 eclipse and has appeared irregularly at other times. There is some indication that it is prominent when the star is unusually bright compared with the mean light-curve.

This paper deals solely with the emission spectrum (1), particularly during eclipse. 47 spectra taken with the 2-prism Cassegrain spectrograph attached to the Radcliffe reflector are available, of which 15, 30 and 2 were at dispersions of 86, 49 and 29 A/mm at Hγ respectively. I am indebted to Dr M. W. Feast for taking 7 of these plates, and to Dr A. J. Wesselink for one. The remainder were taken by the writer.

* CPD $-66°$ 3307′, R.A. $18^h\,15^m \cdot 4$, Dec. $-66°$ 06′ (1950), 10·2 to 12·7 m with strong fluctuations.

2. *Representative elements in emission.*

H. The Balmer lines are strong, Hβ being the strongest bright line in the photographic region. Hγ, Hδ can be resolved into two components with suitable exposures (see following sections).

He I. Well represented.

He II. Outside eclipse, 4686 is the strongest line following Hβ. The Pickering series has not been observed.

C II. 4267 is weakly present on some plates.

C III. 4649 is quite strong.

N III. 4640, 4634, 4097 are present. 4640 and C III 4649 are about equally strong.

[O II]. 3727 is not found on our plates.

[O III]. 5007, 4363, 4959 (in decreasing order of intensity) are strong.

[Ne III]. 3868, 3967 are quite strong.

[S II]. 4068, 4076 appear to be weakly present on a few plates.

Fe II. 4233, 4583, etc., are found on a few plates, especially during eclipse.

[Fe II]. 4244, 4287 are likewise seen occasionally.

3. *Structure of emission lines.*—The lines due to He I, [O III], [Ne III] appear to be quite sharp. On the other hand the Balmer lines have P Cyg structure of Beals' type III (3). Mrs Mayall (1) found violet-displaced absorption on Harvard spectra; a secondary emission peak to the violet of the absorption appears on nearly all Radcliffe spectra. The ratio of intensity R/V varies but has never been found less than 1.

He II 4686 always appears diffuse, and on one plate (29 A/mm) is resolved into two diffuse components of equal intensity; the only other plate at this dispersion is too densely exposed for resolution. Were it not for the intrinsic diffuseness of the lines it seems certain that plates at lower dispersion would have commonly revealed doubling of 4686 similar to that of Hγ and Hδ.

No other lines have been resolved into two components like those of H and of He II 4686. One plate shows a weak companion to the violet of [Ne III] 3868·74 but this is attributed to He I 3868·48.

4. *Separation of emission components.*—The separation of Hγ, Hδ emission components has been measured on 27 plates. Expressed in km/s there is no significant difference between the measured separations on any one plate; for plates where it is possible to measure both separations the mean difference Hγ–Hδ is found to be $+3{\cdot}2 \pm 8{\cdot}7$ km/s. The separation is plotted against phase according to Mrs Mayall's ephemeris in Fig. 1. In view of the marked changes in light from cycle to cycle, different symbols are used to distinguish successive cycles (from phase -300 to $+300$ days). Cycle IV (1957), the only occasion when it was possible to secure observations on both sides of the minimum, has successive points joined.

There is a definite narrowing of the separation during eclipse shown by various cycles. Observations of separation during eclipse are difficult; many spectra available at this phase (when the star is at 12 or 13 m) had to be made at 86 A/mm and the components are consequently unresolved. But there is only one observation in Fig. 1 outside of eclipse (at phase -155d) where the separation was comparable with what is normally found during eclipse.

Fig. 1 also shows a tendency for the separation to be larger after eclipse than before. However, the bulk of the post-eclipse observations refer to cycle II (1954)

when as previously remarked the spectrum 2 (continuum plus super-giant absorption) was peculiarly strong. So far, there is no convincing evidence of a systematic difference in separation before and after eclipse.

The one observation of double He II 4686 (at phase +261d) yielded a separation of 137 km/s as compared with 111 km/s for Hγ and Hδ.

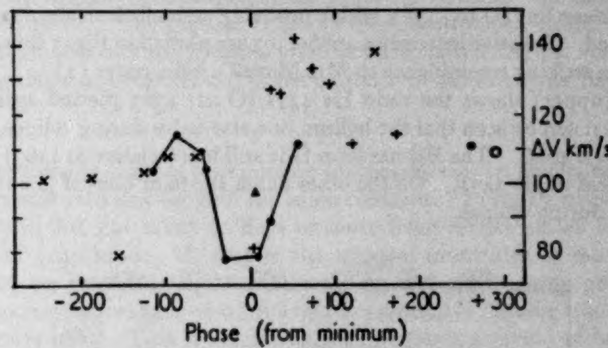

FIG. 1.—*Separation of hydrogen components as function of phase.*

Cycle I: ○ (1953),
II: + (1954),
III: × (1955–56),
IV: ● (1957–58),
V: ▲ (1959).

5. *Intensity ratios of components.*—Visual estimates of intensities of the two hydrogen components have been made. The ratio R/V (on logarithmic scale) is plotted against phase in Fig. 2, where the various cycles are again distinguished.

Despite considerable scatter, which can only be due to observational error in a minor degree, there is a remarkable tendency for R/V to diminish steadily from phase −250 to 0 days; while for nearly 200 days following eclipse R/V is only slightly greater than 1.

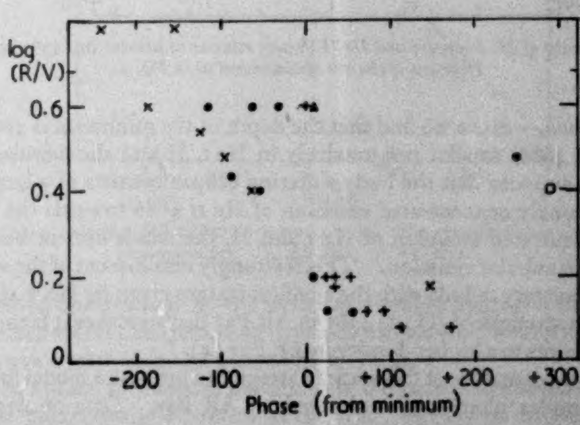

FIG. 2.—*Ratio of hydrogen components as function of phase. Different cycles are distinguished as in Fig.* 1.

6. ***Intensities of emission lines during eclipse.***—The nebular spectrum persists through eclipse. Owing to the longer exposures required at that phase, and to other factors, it is impossible to compare absolute intensities in and outside eclipse from the available material. However, relative intensities of various lines can be estimated with considerable accuracy and these show very interesting changes. Easily the most marked refers to He II 4686 which nearly disappears during eclipse. As a comparison line [O III] 4363, which probably varies little in absolute intensity, has been used. Relative intensities 4686/4363 are plotted in Fig. 3 (lower). This plot shows a striking resemblance to Mrs Mayall's light-curve (1).

Fig. 3 (upper) shows the ratio He 4471/[O III] 4363 plotted against phase, from which it can be seen that the helium line also fades during eclipse, but much less than He II 4686. The Balmer lines fade still less (relative to 4363) perhaps to half the extent of He 4471. On the other hand, the faint lines of [Fe II] are more easily seen during eclipse.

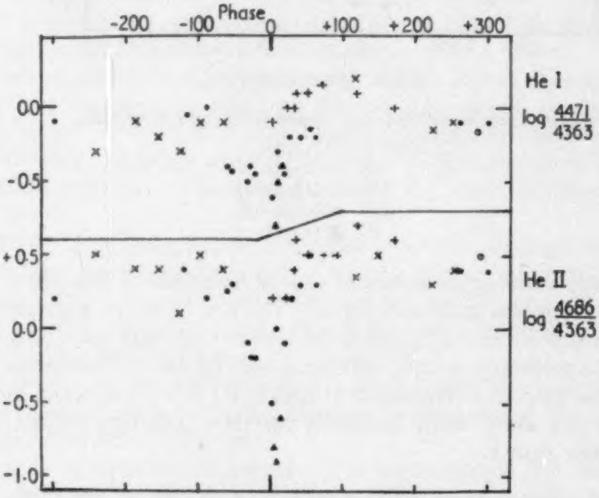

FIG. 3.—*Intensity of He I (upper) and He II (lower) relative to nebular line 4363 against phase. Different cycles are distinguished as in Fig. 1.*

7. *Discussion.*—Since we find that the depth of the minimum is greatest in the light of He II 4686, smaller progressively in He I, H and the nebular lines, it is reasonable to suppose that the body suffering eclipse consists of a large nebulous mass with strongly concentrated emission of He II 4686 towards the centre, less strongly concentrated emission of He I and H, the whole system being diffused with forbidden nebular emission. This is strongly reminiscent of the stratification of ions in a planetary nebula with the smallest images given by He II 4686. However, the great strength of [O III] 4363 in AR Pav distinguishes it from a planetary nebula, and points to a higher density in AR Pav (4).

It would be premature at this stage to attempt to propose a model to account for the many complex phenomena exhibited by AR Pav. Data on the possibility of orbital motion and on the absorption spectra are still too fragmentary. But the behaviour of the double emission components does call for some discussion.

Fig. 2 shows that the red component of the H lines is much stronger than the violet component before eclipse, while for a long time afterwards the two components are nearly equal in intensity. This is rather reminiscent of the eclipse of the gaseous ring surrounding RW Tau studied by Joy (5).

In all the gaseous rings studied by Struve (6) the violet component is weak or invisible at the beginning of eclipse, while at the end of eclipse the same is true of the red component. The gaseous rings are therefore rotating in the same sense as the orbital motions of the binaries. Struve has shown that the velocity V_e in the rotating ring is roughly related to the period P of the binary by the Keplerian relation

$$V_e^3 \propto P^{-1}.$$

It is a remarkable fact that the emission components of AR Pav, outside eclipse, closely obey this relationship with the same constant. From 16 measures of the components in AR Pav taken 60 days or more from minimum we find a mean separation of 114·1 km/s. If, despite the unequal intensities of red and violet components, we treat them as due to receding and approaching portions of a circular gaseous ring, we have 57·0 km/s rotatory motion of the ring (the inclination must be nearly 90°). This is entered in Fig. 4 against a period of 605 days for comparison with other rings (6).

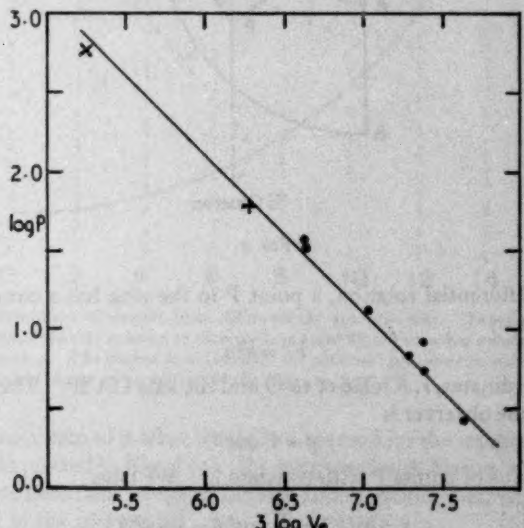

FIG. 4.—*Struve's relation $V_e^3 \propto P^{-1}$ for gaseous rings. The straight line indicates exact proportionality.* × *AR Pav*; + *GG Car.*

Unpublished observations by the writer of doubled hydrogen components in Radcliffe spectra of GG Car (period 62ᵈ) outside eclipse yield a velocity of 124 km/s for V_e. Struve's relation is also obeyed by GG Car.

It is difficult to see why Struve's relation should be so well obeyed since P is the period of the binary while V_e refers to the ring; Struve in discussing this has suggested that the similarity to Kepler's law may be accidental. The relationship appears to imply that not only the various systems have similar masses but that

there is a fixed relationship between the major axis of the binary and the diameter of the ring*.

8. *Eclipse of a differentially rotating ring.*—Many years ago Struve (7) calculated the emission profiles to be expected from a thin circular rotating ring. So far as the writer is aware, the case of a thick ring subject to differential rotation has not been examined†. In the eclipse of such a ring, one expects that at minimum the faster rotating portions would be totally eclipsed while the slow-moving outer portions could remain in view. Hence during minimum the separation of double components might diminish, as we have found to be the case in AR Pav. Since the nebular spectrum appears to pervade the whole system of AR Pav it is logical to postulate a thick rather than a thin ring.

The problem has been examined graphically. In Fig. 5 AA′B′B represents a quadrant of a ring about a central star O. OAB is the direction of the observer (assumed to be in the plane of the ring). Extension of the ring out of the plane of the paper is ignored.

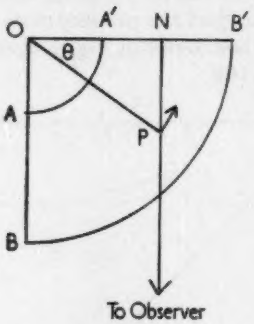

FIG. 5

Assuming differential rotation, a point P in the ring has a circular velocity V around O given by

$$V^2 = k^2/r$$

P has polar coordinates, r, θ relative to O and the axis OA′B′. The radial velocity of P relative to the observer is

$$\rho = V\cos\theta.$$

Consider the locus of points P with constant ρ. We have

$$r = k^2/V^2 = \frac{k^2}{\rho^2}\cos^2\theta = \lambda\cos^2\theta$$

where λ is constant.

In order to study the emission profiles from the ring we assume that

(1) the intensity of radiation from all parts of the ring is the same;

(2) scattering and absorption within the ring can be ignored;

(3) occultation by the central star can be neglected.

* Pointed out by Dr A. J. Wesselink in conversation.

† A useful discussion of profiles in rings, with bibliography, appears in Chap. 3 of *The Galactic Novae*, C. Payne Gaposchkin, North Holland Publishing Co., 1957. The writer of this paper has not had access to *Moving Envelopes of Stars* V. V. Sobolev, Publ. Leningrad State University, 1947.

We first plot loci of constant ρ. Such loci appear as dotted curves in Fig. 6 in equal steps of ρ.

The ring drawn in Fig. 6 has inner and outer radii of 2 and 15 (arbitrary) units respectively. The areas enclosed between successive loci of constant ρ and the boundaries of this assumed ring were measured with a planimeter. With our assumptions, these areas correspond to the intensities in the emission profiles at certain values of ρ; between curves n and $n+1$ we have approximately the intensity of the profile for $\rho = n + \frac{1}{2}$.

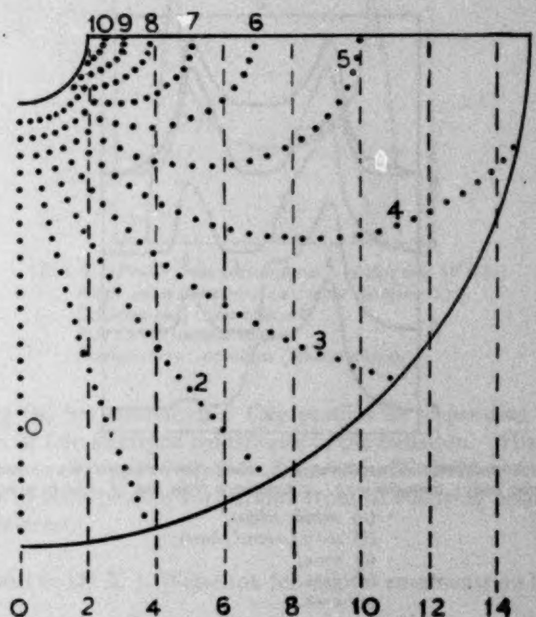

Fig. 6.—*Computation of profile from differentially rotating ring. The dotted curves give loci of constant radial velocity relative to observer ($r = \lambda \cos^2 \theta$), the attached numbers corresponding to the radial velocity ρ. The dashed lines (towards the observer) permit calculation of the emission profile during various stages of an eclipse of the ring (see text).*

The profile so obtained is what would be observed for the integrated light of the whole ring and is plotted in Fig. 7 (a). As compared with Struve's (7) profile for a thin ring we have here faint wings added and rather greater intensity of the central plateau relative to the two peaks.

In Fig. 6 we have also drawn dashed lines in the line of sight, in equal steps of 2 units. The areas enclosed between these lines, the loci of constant ρ, and the boundaries of the ring were also measured with a planimeter. Appropriate summation of the areas so found enables us to calculate the change in profile during the partial phases of the eclipse by a body of diameter $2n$ units. In Fig. 7 (b) – (e) appear profiles at various stages of an eclipse by a sharp edged body with diameter 12 units (as compared with the ring of inner and outer diameters 4 and 30 units).

This model fails to account for the observed effects in AR Pav. At mid-eclipse the faint outer extensions of the profile are extinguished, as expected, but there is a

much more marked extinction of the central plateau; the velocities of the two *peaks* in the profile remain practically unchanged during the eclipse. There is thus no marked narrowing of the separation.

The same result was obtained by assuming a different size for the eclipsing body, for a thinner ring, and even assuming a diffuse eclipsing body. For this diffuse body the assumed transmission was zero for radius 0 to 2 units, 1/3 for radius 2 to 4 units, and 2/3 for radius 4 to 8 units.

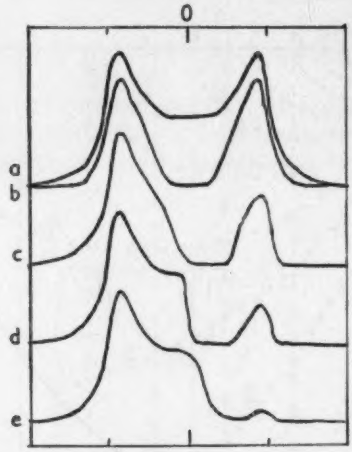

FIG. 7.—*Profiles from differentially rotating ring. Ring: inner diameter = 4; outer diameter = 30. Eclipsing body: diameter = 12, at distance x from line of sight to primary.*

(*a*) *outside eclipse,*
(*b*) $x=0$ *(central phase),*
(*c*) $x=4$,
(*d*) $x=6$,
(*e*) $x=8$.

9. *Expanding ring.*—There appears to be greater hope of accounting for the observed narrowing of emission components in a model based on expansion.

Consider a model in which the gas in a ring is expanding radially from the central star with a velocity proportional to r. The radial velocity relative to the observer is then proportional to $r \sin \theta$, i.e. PN in Fig. 5. The loci of constant radial velocity become straight lines parallel to OA'B'. Their intercepts on the ring can be very easily calculated.

Fig. 8 shows the profile of such an outwardly accelerated ring (full curve) with inner and outer diameters 12 and 15 units respectively. The dashed curve shows how the peaks are extinguished during the eclipse by a body with diameter 8 units. As before, the computed profile is based on the assumption of a uniformly radiating ring. It may be noted that in the case of an outwardly accelerated ring Rosseland (**8**) has shown that re-absorption cannot take place.

The eclipsed profile of Fig. 8 yields peaks with a separation about 70 per cent that of the uneclipsed profile, as is observed in AR Pav. However, it is clear that the predicted profile will remain symmetrical at all phases of the eclipse, provided the motion outwards in the ring is symmetrical. This model will not account for the systematic change in R/V intensity exhibited by AR Pav before and after

eclipse. This latter feature is most easily understood in terms of a rotating model. Perhaps it will be necessary to assume a model combining both rotation and expansion to account for the two observed phenomena—narrowing of separation and change in R/V intensity.

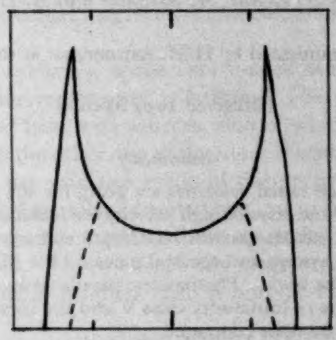

FIG. 8.—*Profile from outwardly accelerated ring* ($V=kr$).
Ring: inner diameter = 12; outer diameter = 15.
Eclipsing body: diameter = 8.
Full curve: outside eclipse.
Dashed curve: in eclipse (central phase).

Rottenberg (9) has computed P Cyg profiles for expanding rings in which recombination of free electrons contributes to the radiation. His Figs. 5 and 6a correspond best to the type of profiles observed in the Balmer series of AR Pav. Computations of such profiles for various types of eclipsing bodies would be of considerable interest.

I am grateful to Dr A. J. Wesselink for helpful comments on this paper.

Radcliffe Observatory,
Pretoria :
1959 *May*.

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Note added in proof.—Greenstein and Kraft (*Ap. J.*, **130**, 99, 1959) have found a relative weakening of He II 4686 during eclipse of Nova DQ Her which seems to be analogous to that reported here in AR Pav.

1959 *December*.

FUNDAMENTAL DATA FOR SOUTHERN STARS (SECOND LIST)

David S. Evans, A. Menzies and R. H. Stoy

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(Received 1959 April 8)

Summary

Newly determined radial velocities are given for 161 stars south of −26°. Several nearby stars and stars of high velocity are included in this list and also nine stars from the galactic clusters NGC 2516 and 2547. Magnitudes and colours on the *B*, *V* system and spectral types on the MK system have been determined for all the stars. Photometric parallaxes have been deduced for 41 non-multiple stars of luminosity class V and the corresponding space co-ordinates and space motions computed.

The fundamental data given in this list form a continuation of, and are intended to be homogeneous with, those given in the First List (1). The great majority of the radial velocity plates have been measured by Dr Evans and Mr Menzies, but Dr Wayman has measured some plates for this list, which, with minor exceptions, closes at 1958 July 1. The systematic corrections to be applied to the measures were evaluated in the usual way by measurement of plates of the reference stars, each plate being measured by all the measurers. The corrections in km/s that were actually applied to the measures are as follows:—

	"a"		"b"			"c"		
	DSE	AM	DSE	AM	PAW	DSE	AM	PAW
Before 1955 June 1	+1·7	+1·7	+0·7	+0·2	..	−1·2	−2·5	..
1955 June 1—1956 May 31	..	..	+0·5	+0·7	..	−0·9	−0·5	..
1956 June 1—1957 May 31	+1·3	..	−0·3	+0·2	..	−1·9	−1·9	..
1957 June 1—1958 May 31	..	..	−0·2	+0·4	+0·4	−1·9	−1·3	−0·9

Several of the reference stars have now been observed so many times that greater weight should be attached to the results of these new observations than to the velocity given in the Mt Wilson Catalogue. The velocities of the reference stars have therefore been re-examined and what is hoped to be a system with a greater degree of internal consistency has been derived by forming weighted means of the catalogue values and the new results. In doing this all the Mt Wilson Catalogue plates were weighted as 4, the new a-dispersion plates as 4, b-dispersion plates as 3, and c-dispersion plates as 1. The results are shown in Table 1 in which the last three columns give for the three dispersions the actual number of new plates used in this examination. The weighted mean difference between these new adopted values and the original catalogue values is 0·00 km/s.

The magnitudes and colours given in Table II were, in general, derived from observations made with the Cape Astrographic Refractor or from a series of three-colour observations made with the Victoria Refractor which has been described

elsewhere (2). In either case, each star has received a minimum of four observations on separate nights and the probable errors of the resulting magnitudes and colours as deduced from the internal agreement are $\pm 0^{m}\cdot 005$ and $\pm 0^{m}\cdot 004$ respectively. The diaphragm of the Astrographic photometer has a diameter of 90″ and that of the Victoria photometer a diameter of 45″ so that it has not been possible to measure individual magnitudes and colours for a number of stars with nearby companions.

The photometric parallaxes, space coordinates and space motions given in Table III have been derived exactly as before. The proper motions given are the weighted mean of at least two sources, one of which was usually the Albany General Catalogue, and the other one of the more recent photographic catalogues. For some of the stars use was also made of the proper motions derived in the course of trigonometrical parallax observations. For only one star in Table III, No. 481, does the photometric parallax differ appreciably from a well determined trigonometrical parallax. If we accept the trigonometrical value, this star is apparently 1·3 magnitudes brighter than a normal F8 V star having the same measured colour.

We would once again like to express our indebtedness to the Union Astronomer, Dr W. S. Finsen, who has kindly provided up-to-date information from the Union Observatory card catalogue for the double stars in this list.

Table I

No.	HD	M.W.C.		Cape		Adopted Vel.	a	b	c
		Vel.	Wt.	Vel.	Wt.				
R 1	693	+14·8*a*	52	+13·7	21	+14·5	0	7	0
R 2	1581	+ 8·7*a*	68	+ 8·0	12	+ 8·6	2	0	4
R 3	4128	+13·1*a*	280	+13·1	32	+13·1	6	2	2
R 4	20794	+86·8*a*	60	+87·5	44	+87·1	3	8	8
R 5	80170	0·0*a*	24	− 0·5	53	− 0·3	2	10	15
R 6	101021	+ 3·4*b*	12	+ 2·5	38	+ 2·7	2	6	12
R 7	114837	−65·0*a*	36	−64·3	43	−64·6	2	7	14
R 8	157457	+17·6*a*	24	+16·9	90	+17·0	4	17	23
R 9	171391	+ 6·6*a*	32	+ 7·4	46	+ 7·1	2	10	8
R10	203638	+22·0*a*	40	+22·1	81	+22·1	6	12	21
R11	223647	+14·5*b*	12	+13·3	30	+13·6	3	6	0

TABLE II

No.	HD	(1950) R.A.	(1950) S. Dec	*V*	*B–V*	Spec.	Vel. (km/s)	Pl.
		h m	° ′					
340	11112	01 46·2	41 45	7·14	+0·64	G4 V	+35·7±0·4	5
341	15481	02 26·3	42 39	8·22	+0·44	F6 IV	+10·3±0·8	5
342	18169	51·8	41 27	8·43	+0·41	F5 V	+39·0±0·6	4
343	20280	03 12·6	26 38	9·13	+1·24	K7 V	+14·9±1·0	6
344	22946	37·6	42 55	8·22	+0·52	F8 V	+18·9±0·6	4
345	26151	04 05·3	27 33	8·49	+0·83	K0 V	− 1·8±1·2	5
346	26770	11·0	28 40	7·46	+0·50	G0 V	+14·0±0·9	4
347						G0 V	+18·6±0·6	4
348	30684	46·0	46 41	8·12	+0·81	G8 V	+43·0±0·7	4
349	36519	05 28·6	43 37	7·73	+1·49	K3 III	+51·9±0·5	5
350		36·8	46 08			K5 V	+13·5±0·9	4
351	39091	41·1	80 31	5·64	+0·59	G3 IV	+ 9·1±0·5	4
352	53705	07 02·4	43 32	5·28	+0·65	G3 V	+86·4±0·4	5
353	59468	26·2	51 18	6·75	+0·70	G5 IV–V	+ 3·4±0·3	4
354	−60°945	55·9	60 42	8·54	+0·03	B9 III	+18 ±2	4
355	−60°947	56·0	60 28	8·09	−0·04	B8 III	+17 ±3	6
356	65869	56·5	60 38	7·74	+0·02	B9 V	+20 ±2	5
357	65950	56·9	60 47	6·88	−0·03	B9 III	+25·0±1·4	5
358	65987	57·2	60 29	8·01	−0·07	A0p Sr	+23 ±2	5
359	−60°980	57·3	60 41	6·69	+1·29	K1 III	+21·6±0·9	5
360	66194	58·0	60 41	5·80	−0·12	B3en	+22 ±5	4
361	68451	08 08·8	48 53	7·32	−0·12	B2 III	+20·1±1·5	4
362	68608	09·5	49 08	7·88	−0·09	B5 III	+14·4±1·2	4
363	70642	19·7	39 33	7·17	+0·70	G6 IV–V	+48·1±0·4	4
364	71701	22·2	77 19	4·38	+1·16	K0 III–IV	+21·4±0·3	6
365	71805	25·6	52 32	6·50	+0·39	F6 V	+ 7·8±0·5	4
366	73744	33·5	76 45	7·61	+0·59	G0 V	+44·4±0·8	5
367	74868	43·1	44 22	6·57	+0·55	G3 IV	+21·1±0·3	4
368	79837	09 04·4	85 28	5·42	+0·30	F0 III	− 4·7±1·8	4
369	79416	10·7	43 24	5·55	−0·10	B8 V	+15·4±1·7	5
370	81044	20·2	31 57	8·84	+0·79	K0 V	+26·9±1·3	4
371	81575	23·3	43 46	6·4	+1·6	M5 III	+39·9±0·7	4
372	82455	28·8	47 23	8·65	+0·65	G5 V	+36·6±1·0	8
373	83443	35·3	43 03	8·21	+0·81	K0 V	+27·6±0·5	9
374	89090	10 13·8	28 22	7·22	+0·52	G1 V	+32·4±0·6	5
375	90589	23·4	73 47	3·99	+0·35	F3 IV–V	− 5·3±0·7	4
376	90519	24·1	45 17	7·82	+1·38	K1 III	− 8·6±0·6	4
377	90520	24·1	45 19	7·51	+0·62	G3 V	+17·6±0·3	5
378	90559	24·3	43 09	8·17	+1·23	K1 III–IV	+15·3±0·6	4
379	90740	25·6	44 05	7·11	+0·89	G5 III	− 2·4±0·4	5
380	93144	42·3	55 17	8·18	+1·31	K1 III	−10·1±0·9	4

TABLE II (*cont.*)

No.	HD	(1950) R.A.	S. Dec	*V*	*B–V*	Spec.	Vel. (km/s)	Pl.
		h m	° ′					
381	93173	10 42·6	43 42	9·00	+0·67	G5 V	+27·2±0·7	4
382	94906	54·5	30 56	7·42	+0·33	F2 V	− 3·3±0·5	6
383	98220	11 15·2	33 16	6·85	+0·48	G0 IV–V	+16·7±0·7	5
384	99279	22·5	61 22			M0 V	+ 4·5±0·8	5
385	100901	33·6	72 34	6·54	+1·18	K1 IV	+ 2·5±0·6	4
386	101266	36·4	45 05	9·29	+0·65	G5 IV	+20·6±0·5	5
387	101493	38·1	42 53	8·62	+0·47	F5 V	+ 3·5±1·3	4
388	102438	44·8	30 00	6·48	+0·68	G5 V	+11·8±0·8	5
389	102596	46·0	47 17	8·99	+1·43	K3 III	−19·0±1·1	4
390	102769	47·3	45 44	7·60	+1·32	K1 III	+16·1±0·9	4
391	108309	12 24·2	48 38	6·25	+0·67	G5 IV–V	+29·3±0·3	4
392	109842	35·5	46 50	8·09	+0·49	F6 IV–V	+ 6·1±1·5	5
393	110253	38·4	43 50	6·76	+1·28	K3 III	+ 9·1±0·6	4
394	110619	41·0	37 26	7·52	+0·66	G5 V	−22·5±0·7	4
395	110838	42·6	47 54	6·66	+1·16	K1 III	+26·2±0·3	6
396	111417	46·7	45 33	8·33	+1·41	K3 IV	−16·0±0·5	5
397	111535	47·6	46 57	7·97	+0·45	F6 IV	−29·8±1·2	6
398	111775	49·3	47 49	6·32	+0·03	A0 II	− 2·2±1·0	5
399	111777	49·4	56 17	8·45	+0·60	G3 V	+ 3·3±0·7	4
400	112213	52·5	42 39	5·46	+1·69	M0 III	− 6·9±0·4	6
401	112437	54·3	46 56	8·18	+1·24	K0 III–IV	+17·7±0·6	5
402	112685	56·3	45 42	7·87	+0·30	F3 IV	+ 9·6±1·1	6
403	113537	13 02·2	46 51	6·46	+0·40	F5 III	− 1·4±1·0	4
404	116064	19·0	39 04	8·79	+0·46	F0p	+141·9±1·3	4
405	117939	31·5	38 38	7·30	+0·66	G4 V	+80·6±0·6	4
406	118646	35·9	29 18	5·83	+0·39	F6 IV–V	+ 4·2±0·5	11
407	119985	44·6	45 49	8·54	+0·63	G3 IV–V	+33·5±0·8	6
408	120237	46·0	35 27	6·52	+0·55	G3 IV–V	+ 2·1±0·1	4
409	121141	51·6	47 53	7·18	+0·34	F2 V	−16 ±2	6
410	121746	55·3	48 13	7·16	+0·46	F5 IV	−24·5±1·0	4
411	123682	14 07·1	44 45	8·28:	+0·69:	G5 V	+48·0±1·1	6
412	126525	24·2	51 43	7·82	+0·68	G5 V	+11·8±0·5	4
413	128674	36·8	56 49	7·37	+0·66	G5 V	+31·8±0·5	4
414	129642	41·8	49 42	8·38	+0·95	K3 V	− 4·6±0·6	4
415	130265	45·7	58 55	8·53	+0·63	G3 V	−72·3±0·6	6
416	130551	47·6	60 44	7·17	+0·42	F8 V	+32·1±0·4	6
417	130807	48·4	43 22	4·32	−0·12	B8 V	+ 7·4±1·1	10
418	131078	49·7	46 26	8·15	+0·70	G5 V	−17·1±0·5	5
419	131168	50·0	45 39	6·98	−0·07	B3 Ve	+ 1 ±4	7
420	132785	59·3	48 20	9·31	+0·37	F0 Vn	− 8·5±2·2	4

TABLE II (*cont.*)

No.	HD	(1950) R.A.	S. Dec	V	$B-V$	Spec.	Vel. (km/s)	Pl.
		h m	° ′					
421	129723	15 01·4	87 57	6·48	+0·30	F0 III	−14·5±0·8	4
422	133612	03·8	47 48	8·89	+0·92	K0 V	−70·5±0·6	4
423	133955	05·5	45 05	4·08	−0·20	B3 V	+ 8·8±1·3	11
424	134505	08·7	51 55	3·45	+0·90	G8 III	−10·8±0·4	5
425	137676	25·7	49 47	7·66	+0·77	G5 V	−42·7±0·3	5
426	138690	31·8	41 00	2·77	−0·23	B3 Vn	−16 ±3	7
427	139465	36·7	44 52	7·40	+1·28	K4 III	−22·1±0·5	4
428	142709	54·3	42 29	8·06	+1·13	K5 V	+35·6±0·9	5
429	143120	56·9	45 19	7·53	+0·73	G5 IV	−23·6±0·8	4
430	143138	57·1	47 45	8·70	+1·41	G6 III	−20·6±0·6	5
431	143234	57·6	45 13	8·69	+0·25	A0 V	−13 ±3	5
432	144899	16 06·7	47 47	8·97	+0·65	G5 IV	− 2·5±0·9	4
433	146800	16·6	48 06	8·91	+0·95	K3 V	+ 1·2±0·6	6
434	149606	34·2	40 46	8·97	+0·96	K2 V	− 2·5±0·7	4
435	149640	34·5	44 12	7·93	+1·19	K0 IV	−29·6±0·7	5
436	151849	48·4	45 23	8·44	+0·51	F2 IV	−16·5±1·2	4
437	151967	49·8	57 50	5·92	+1·61	M0 III	−40·7±0·4	4
438	152236	50·5	42 17	4·3	+0·73	B0.5Ie	−34·0±1·2	4
439	152250	50·8	44 49	7·40	+0·38	F0 Vn	−16 ±2·5	5
440	152798	54·0	45 16	8·77	+0·57	G3 IV	+ 8·4±0·9	6
441	153026	55·4	39 29	8·33	+1·16	K5 V	+42·5±0·7	4
442	154088	17 01·3	28 31	6·57	+0·83	G8 IV–V	+14·2±0·4	4
443	154810	06·2	45 34	8·14	+0·48	F8 V	−26·0±0·9	6
444	155203	08·6	43 11	3·34	+0·40	F0 IVn	−26 ±3	4
445	155185	08·7	46 29	9·18	+0·85	K0 V	−63·0±1·0	4
446	155885	17 12·3	26 32	4·38	+0·84	K1 V	0·0±0·4	5
447	−46°11370	15·3	46 35			M0 V	+20·8±0·8	4
448	160043	35·9	28 23	7·71	+0·41	F6 V	− 3·5±0·7	5
449	162021	47·3	42 19	6·67	+1·04	K0 III	−22·9±0·3	5
450	162619	50·5	47 25	8·75	+1·23	K1 IV	−36·8±0·9	5
451	166006	18 07·6	47 31	6·06	+1·20	K1 III–IV	−14·9±0·4	4
452	172144	37·1	44 13	7·39	+0·54	G2 IV	+ 4·7±0·5	5
453	172462	38·9	44 17	8·75	+0·34	F5 IV	+41·5±1·4	5
454	173182	42·4	42 35	7·50	+1·05	G8 III	−14·1±0·9	6
455	173183	42·5	42 37	6·92	+0·40	F2 IV	− 2·2±0·7	4
456	173697	45·2	45 19	7·26	+0·99	G5 III–IV	−28·3±0·5	5
457	173791	45·8	45 52	5·80	+0·89	G6 IV	+ 9·7±0·4	5
458	174386	48·7	44 24	8·17	+0·40	F2 V	−22·1±1·6	4
459	174978	51·7	43 57	9·18	+0·47	F6 IV	− 4·0±1·1	4
460	175219	52·7	42 47	5·35	+1·00	G6 III–IV	−21·0±0·4	5

TABLE II (*cont.*)

No.	HD	(1950) R.A.	S. Dec	V	$B-V$	Spec.	Vel. (km/s)	Pl.
		h m	° ′					
461	176427	18 58·8	44 13	8·34	+0·32	F2 IV	+14·4±1·1	4
462	177565	19 03·5	37 53	6·14	+0·70	G5 IV	+58·5±0·3	4
463	177688	04·1	43 00	9·01	+0·51	G0 IV	−10·6±0·8	4
464	178395	06·8	42 49	9·29	+1·09	K0 IV	−26·9±0·6	4
465	181743	20·2	45 10	9·64	+0·46	Fp	+18 ±1·2	4
466	185993	40·2	44 15	7·28	+1·24	K3 III	+ 8·4±0·3	7
467	186012	40·3	43 34	9·10	+0·36	F0 V	− 9·5±1·9	6
468	186682	44·3	45 50	7·24	+0·14	A3 V	−11·2±0·9	4
469	187369	48·0	42 13	7·85	+0·57	G2 IV	−30·7±0·4	6
470	188011	51·6	46 55	7·69	+1·09	K0 III–IV	+37·7±0·8	4
471	188903	55·7	41 58	8·27	+0·55	G2 V	− 5·2±1·2	4
472	189140	57·0	43 11	6·12	+1·68	M0 III	−33·5±0·4	8
473	189585	59·1	44 22	8·89	+0·97	G8 IV	+ 5·6±0·7	5
474	189631	59·3	41 33	7·56	+0·29	A9 V	− 6·9±1·0	4
475	190309	20 02·7	44 29	7·86	+1·13	K1 III	−57·0±0·4	6
476	190879	05·5	47 13	6·45	+1·50	K5 III	−57·6±0·8	4
477	191190	07·1	46 53	6·81	+1·16	K1 IV	−50·6±0·3	4
478	191935	10·6	44 19	8·39	+0·43	F8 IV	−10·3±0·9	4
479	192071	11·4	44 13	8·40	+0·61	G3 V	−15·5±0·8	6
480	192961	16·1	46 35	8·72	+1·18	K5 V	+29·9±0·9	4
481	196378	35·9	60 43	5·11	+0·51	F8 V	−31·2±0·5	4
482	196227	37·1	76 43	7·66	+0·59	G1 V	+21·1±0·9	4
483	196829	38·1	42 19	6·30	+1·61	M3 II	−22·9±0·5	4
484	198009	45·8	46 50	7·81	+1·22	K0 III–IV	−61·7±0·5	6
485	199190	53·3	69 46	6·87	+0·61	G5 IV	−34·3±0·4	4
486	200361	21 01·2	44 40	9·33	+0·66	G5 V	− 7·8±0·9	6
487	200553	02·2	43 43	7·20	+0·93	G8 IV	−14·7±0·3	4
488	200525	04·2	73 22	5·66	+0·58	G3 IV	− 9·1±0·6	4
489	201245	06·4	44 25	6·51	+1·17	K1 III	+ 2·0±0·4	5
490	202103	12·2	53 28	5·73	+0·20	A7 V	−12·9±1·1	12
491	202628	15·2	43 33	6·74	+0·63	G5 V	+10·7±0·3	4
492	203850	23·3	56 21	8·64	+0·92	K3 V	−42·1±0·7	4
493	207852	50·1	47 04	7·42	+0·56	G0 IV–V	+31·2±0·6	4
494	208323	53·6	46 43	7·44	+0·37	F5 IV–Vn	−19·9±0·8	5
495	208627	55·5	44 18	6·54	+0·89	G8 IV	+13·4±0·3	5
496	208710	56·2	46 35	7·58	+1·25	K3 III	−27·0±1·0	4
497	210918	22 11·6	41 37	6·24	+0·64	G5 V	−19·8±0·3	4
498	212038	19·4	51 03	8·73	+0·84	K0 V	− 0·4±0·7	4
499	214065	33·6	46 43	9·25	+1·21	K1 III	+ 3·8±1·4	4
500	222741	23 40·8	41 53	8·40	+0·47	F8 V	+ 8·1±0·9	5

Notes

345 Velocity possibly variable.
346, 7 ADS 3069 Bi. 8·1 : 8·3, 2″·03, 44°·3 (1955·01). 346 is sp, 347 nf.
350 GC 7049. Fainter component of binary formed with No. 71.
351 π Men.
352 HR 2667. Joint magnitude and colour with HR 2668. Tri. AB 5·8 : 6·9, 20″·53, 123°·1 (1930·52), AC 5·8 : 9·3, 184″·8, 334°·4 (1900·60).
354–360 Members of the open cluster NGC 2516. The mean velocity of these stars is +21 km/s. Magnitudes and colours derived from *Ap.J.*, **121,** 628.
354 h 4027 Quadruple AB 9·1 : 9·6, 9″·42, 114°·7 (1917·23), AC 9·1 :?, 29″·12, 283°·6 (1872·18), AD 9·1 : 11·1, 46″·44, 277°·6 (1872·18).
357 Interstellar H and K, +17 km/s.
358 Interstellar K, +18 km/s.
360 Hβ emission, +28 km/s. Difficult spectrum. The discrepancy with the b quality Lick velocity of −2·7 km/s is unexplained.
361 Member of NGC 2547. Interstellar K, +11 km/s.
362 Member of NGC 2547. Interstellar K, +17 km/s.
364 θ Cha. Bi. 4·4 : 7, 31″·3 :, 250°·1 (1911·20).
365 B 1606 Bi. 7·2 : 7·4, 0″·18 fast moving, period 14 years, nodal passages 1954 and 1964. Both stars on the slit. Mean date of velocity observations 1956·47.
368 ζ Oct.
369 ϕ 317, h 4188 Tri. AB,C 6·0 : 6·7, 2″·66, 282°·3 (1947·11). AB interferometric 6·5 : 7·0, 0″·116, 130°·1 (1951·3), too close for measurement 1952–56. Magnitude and colour refer to combined light.
370 Velocity possibly variable.
371 Brightness varies through at least 0·1 magnitude.
372 Velocity possibly variable.
375 This star is designated as " SB " in the Mt Wilson Catalogue. This seems to be an incorrect transcription from the literature. The only claim made was that the velocity was variable. Even this seems doubtful.
378 I 1200 Bi. 8·2 : 13, 2″·87, 142°·7 (1927·34).
384 Fainter component of binary formed with No. 131. H and K in emission, −3 km/s.
387 h 4464 Bi. 9·0 : 10·0, 10″·66, 159°·0 (1929·43). The radial velocity and spectral type refer to the brighter star, the magnitude and colour to the combined light. Spectrum lines are weak.
391 Common proper motion with a tenth magnitude star $0^{m}\cdot 8$ pr and 3′ n.
397 Velocity possibly variable.
404 A high velocity dwarf. The peculiarities of this spectrum are quite striking. There are few prominent lines. H and K are strong and broad, while the Balmer lines are of moderate strength and narrow. Apart from elusive shadows, the only remaining lines are a broad faint line near 4000, 4045 (weak), 4063, 4066 (very weak), 4071 (weak), 4132 (a faint blend), 4144, 4202 (faint blend), 4215 (very weak), 4226, 4250 (faint), 4271, faint shadows near 4290 and 4300, 4325, 4383 and 4415. Excluding blends, the lines are narrow.
408 Bi. 6·6 : 9·6, 11″·54, 355°·5 (1938·70).
409 Difficult spectrum, broad lines.
411 Common proper motion with fainter star 57″ nf it which interferes with the measures of the magnitude and colour by the Astrographic photometer.
417 o Lup ϕ 319 4·9 : 5·3, 0″·129, 125°·0 (1955·64).
418 I 952 Bi. 8·3 : 10·3, 0″·71, 270°·4 (1956·41).
419 Δ 171 Bi. 7·1 : 9·1, 17″·54, 226°·3 (1932·55). Companion noted as red by Herschel and van den Bos. A very difficult spectrum with double reversal of Balmer lines. Velocity possibly variable.
420 Broad lines.
423 λ Lup ϕ 219 Bi. 4·7 : 4·9, 0″·23, 170°·6 (1956·41). Difficult spectrum. An early suspicion of variable velocity was not confirmed.
424 ζ Lup. Δ 176 Bi. 3·5 : 7·9, 72″·3, 248°·7 (1938·5).

426 γ Lup. h 4786 Bi. 3·4: 3·6, 0″·40, 285°·2 (1956·41); period 147 years, a = 0″·59. Difficult spectrum; velocity possibly variable.

430 Spectrum difficult to classify, but star is very red for any possible classification.

438 Interstellar H and K, −11 km/s. Hβ emission, +1 km/s.

441 I 580 Bi. 8·5: 10·5, 3″·88, 9°·0 (1931·32).

444 η Sco.

446 36 Oph. 5·29: 5·33, 4″·37, 166°·0 (1955·50). The magnitude and colour refer to the joint light while the radial velocity and spectrum refer to the south component only. H and K lines in emission, +2 km/s. The north component is No. 207. There are also faint companions at distances 38″·6, 208″ and 732″. S. Archer (*M.N.*, **117**, 640, 1957), has given the following velocities for these stars:—No. 207, +0·2 km/s (6 plates); No. 446, −0·5 km/s (6 plates).

447 This is the second component of the multiple star 41 Ara of which No. 209 is the principal component.

451 B 1879 Bi. 6·1: 12, 1″·70, 266°·4 (1934·35).

465 This spectrum somewhat resembles that of No. 404 but the peculiarities are not so outstanding. The line near 4000 is not present and in general the lines are broader and somewhat more numerous. The line 4077 is present.

467 h 5139 Bi. 9·2: 11·7, 15″·25, 130°·3 (1943·72). Spectrum lines broad and occasionally suspected of being double.

477 I 123 Bi. 6·9: 11·5, 8″·19, 184°·5 (1954·65).

481 ϕ^2 Pav.

488 I 379. This star has been independently discovered as a very close binary in 1898 and 1932, but there are many negative observations including interferometric ones in 1952 and 1953. There is a 13th magnitude companion 7″·86 away in p.a. 130°·7 (1931·06).

490 ϕ 329 Bi. 6·5: 6.5, 0″·164, 70°·5 (1954·79). Velocity probably slightly variable.

Note:—See Evans, Menzies and Stoy (*M.N.*, **117**, 534, 1957): Nos. 234 and 235 are, respectively, the sp and nf components of γ Cr A. The velocities given, viz., −50·8 km/s and −53·2 km/s (mean epoch 1953·9), are to be compared with those of Archer (*loc. cit.*), −52·4 km/s and −54·3 km/s, (mean epoch 1956·43).

TABLE III

No.	$B\text{–}V$	M_V	π_{sp}	π_{tr}	μ_α	μ_δ	x	y	z	u	v	w
Type F5 V												
			″		″	″						
342	+0·41	3·73	0·012		+0·048	+0·018	+16	−40	−76	+ 25	− 27	−26
Type F6 V												
448	+0·41	3·73	0·016		−0·093	−0·067	−63	0	0	+ 4	− 32	+13
Type F8 V												
344	+0·52	4·58	0·019		−0·040	−0·117	+13	−32	−42	− 26	− 19	−18
416	+0·42	3·82	0·021		+0·056	+0·107	−34	−32	− 2	− 39	− 6	+14
443	+0·48	4·32	0·017		−0·019	−0·098	−55	−18	− 5	+ 33	− 15	−10
481	+0·51	4·53	0·077	—	+0·294	−0·565	− 9	− 4	− 8	+ 45	− 20	+10
			—	0·041			−17	− 8	−15	+ 64	− 47	+ 1
500	+0·47	4·24	0·015		+0·110	+0·051	−21	− 8	−65	+ 35	0	−20
Type G0 V												
366	+0·59	4·86	0·028		−0·177	+0·172	−11	−31	−13	+ 26	− 50	−24
Type G1 V												
374	+0·52	4·58	0·030		−0·243	+0·046	+ 2	−30	+14	+ 37	− 34	− 2
482	+0·59	4·86	0·028		+0·070	−0·158	−22	−21	−19	+ 8	− 33	−13
Type G2 V												
471	+0·55	4·73	0·020		+0·011	−0·094	−44	− 1	−26	+ 9	− 21	− 3
Type G3 V												
377	+0·62	4·91	0·030		−0·089	+0·065	− 5	−32	+ 6	+ 15	− 19	+ 5
399	+0·60	4·88	0·019	0·011	−0·680	−0·226	−28	−43	+ 5	+137	− 99	−51
415	+0·63	4·92	0·019		−0·128	−0·047	−39	−36	0	+ 76	+ 24	+ 4
479	+0·61	4·90	0·020		−0·045	+0·012	−41	− 4	−28	+ 6	+ 2	+18
Type G4 V												
340	+0·64	4·94	0·036	0·060	+0·390	+0·154	+ 1	− 9	−27	+ 52	− 29	−26
405	+0·66	5·00	0·035	0·035	+0·450	−0·390	−18	−20	+11	−104	− 40	−29
Type G5 V												
372	+0·65	4·97	0·018	0·024	−0·244	+0·060	− 2	−55	+ 3	+ 58	− 40	−29
381	+0·67	5·05	0·016		−0·067	−0·070	−10	−59	+15	+ 2	− 34	−20
388	+0·68	5·10	0·053	0·058	−0·270	−0·240	− 5	−16	+10	+ 10	− 28	−17
394	+0·66	5·00	0·031	0·032	−0·620	−0·212	−15	−25	+13	+ 83	− 43	−39
412	+0·68	5·10	0·029	0·052	−0·302	+0·031	−26	−23	+ 5	+ 24	− 39	+25
413	+0·66	5·00	0·034	0·043	+0·394	−0·309	−21	−20	+ 1	− 46	0	−60
425	+0·77	5·64	0·039	0·052	−0·218	−0·091	−22	−14	+ 2	+ 52	0	+ 3
486	+0·66	5·00	0·014		+0·076	−0·050	−53	− 5	−50	+ 25	− 16	−13
491	+0·63	4·92	0·043		+0·230	+0·017	−16	− 1	−17	+ 11	+ 1	−25
497	+0·64	4·94	0·055	0·040	+0·558	−0·790	−10	0	−15	+ 41	− 74	− 2
Type G8 V												
348	+0·81	5·92	0·036		−0·056	−0·073	+ 7	−21	−18	0	− 30	−33

TABLE III (*cont.*)

No.	$B-V$	M_v	πsp	πtr	μ_α	μ_δ	x	y	z	u	v	w
Type K0 V												
345	+0·83	6·04	0·032		−0·060	−0·282	+15	−15	−22	− 37	− 18	−11
370	+0·79	5·78	0·024	0·025	−0·618	+0·270	+ 7	−39	+ 9	+129	− 14	−39
373	+0·81	5·92	0·035		+0·022	−0·135	0	−29	+ 3	− 14	− 29	− 9
422	+0·92	6·50	0·033		−0·097	−0·070	−24	−17	+ 4	+ 67	+ 26	−11
445	+0·85	6·16	0·025	0·016	−0·058	−0·068	−38	−13	− 4	+101	− 83	−60
498	+0·84	6·10	0·030	0·021	+0·160	−1·070	−18	− 6	−27	+ 11	−166	+32
Type K2 V												
434	+0·96	6·67	0·035	0·031	−0·374	−0·376	−28	− 9	+ 1	+ 25	− 68	+ 4
Type K3 V												
414	+0·95	6·63	0·045	0·026	−0·656	−0·378	−17	−14	+ 3	+ 53	− 58	− 6
433	+0·95	6·63	0·035	0·033	−0·421	−0·790	−26	−13	0	+ 49	−106	−36
492	+0·92	6·50	0·037	0·043	+0·665	+0·138	−18	− 7	−19	+ 88	+ 25	−32
Type K5 V												
428	+1·13	7·02	0·062	0·055	−0·256	−0·202	−14	− 7	+ 2	− 22	− 37	+ 6
480	+1·18	7·15	0·049	0·058	−0·372	−0·097	−16	−11	−11	− 42	− 18	+11
Type K7 V												
343	+1·24	7·39	0·045		+0·200	+0·086	+ 9	− 7	−18	+ 24	− 13	0

In addition to those given in Table III, trigonometrical parallaxes from the Yale Catalogue are available for the following stars:

No.	π		No.	π		No.	π
	″			″			″
351	+0·038		391	+0·036		442	+0·066
352	+0·044		404	+0·002		444	+0·063
353	+0·052		407	+0·025		446	+0·183
363	+0·055		408	+0·030		447	+0·125
364	+0·027		411	+0·014		460	+0·004
367	+0·028		418	+0·022		462	+0·056
375	+0·079		424	+0·036		465	+0·003
383	+0·016		429	+0·040		485	+0·030
384	+0·085		441	+0·069		488	+0·040

The Royal Observatory,
Cape of Good Hope:
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AN EXPERIMENTAL INVESTIGATION OF THE EFFECTS OF CONFUSION IN A SURVEY OF LOCALIZED RADIO SOURCES

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Summary

Radio source surveys have been made using both a total power equipment and an interferometer. It is shown that in the regions of sky common to the surveys there are considerable discrepancies between the two lists of sources and the majority of these discrepancies are probably due to confusion effects. A comparison of the lists of sources enables an estimate to be made of the reliability of a confusion-limited survey. This limit seems to occur at about one source per 25 beam areas. Application of this limit to other surveys suggests that the Cambridge 2C survey is severely resolution-limited, and that the apparent increase in the density of sources with distance deduced from this survey must therefore be considered extremely doubtful. The Sydney survey which is compatible with an isotropic distribution of sources seems, however, to be free from serious resolution limitations. The conclusions drawn from this latter survey would therefore appear to be more reliable than those drawn from the Cambridge 2C survey.

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1. *Introduction.*—A fundamental limitation to the number of radio sources observable with a given aerial system is set by the finite solid angle of reception of the aerial beam. In any survey, whether it be a total power survey or an interferometer survey, errors will tend to occur whenever two or more sources are present simultaneously in the aerial beam. If the receiving system is a total power system the contributions from the sources will add, and they may be interpreted as a single source of greater intensity than each of the individual sources, or the sources may be resolved but the intensities and positions may be subject to considerable errors.

With an interferometer there will be beating between the fringe patterns of each source and the appearance of the record will depend on the relative phases of these patterns. Thus, the interpretation of the record in a confused region will in practice be different from the interpretation which would be placed on a total power record covering the same region. It is therefore to be expected that, at the intensity level at which the confusion effects become serious, a survey of a given region of sky using an interferometer will give rise to different results to a survey made using a total power equipment. A comparison of the results obtained from a total power survey and an interferometer survey should therefore enable an estimate to be made of the reliability of a survey which is resolution limited. This paper describes the result of such a comparison.

2. *The total power survey.*—The total power equipment was similar to the system which was used at 158 Mc/s by Hanbury Brown and Hazard (1). The aerial system was a paraboloid of 218 ft diameter and 126 ft focal length and at a frequency of 92 Mc/s the beam width was 3 degrees between half-power points and the power gain 1390 over a half wave dipole. The beam was directed to different declinations by tilting in the north–south plane the mast which supports the

primary feed. The maximum angle of tilt was determined by the loss of gain and distortion of the beam with feed tilt and this restricted the available field of view of the aerial to the area of sky between declination 26°N and 80°N.

The design of the receiving equipment was based on that described by Ryle and Vonberg (2) in which the power received by the aerial is balanced continuously against that from a standard generator. The receiver had a pre-detector bandwidth of 0·5 Mc/s, a post-detector time constant of 10 seconds and a noise factor of 2·5. It was connected to the aerial by a coaxial cable with an attenuation of 2·2 db. During the daytime the minimum flux detectable was limited by external interference but at night it was possible to approach the theoretical limit of about 3×10^{-26} watts $m^{-2}(cps)^{-1}$ set by the noise fluctuations in the receiver.

For the examination of a particular declination strip the aerial beam was set at a fixed elevation corresponding to the required declination and the received power was recorded for the required period of right ascension. The region was surveyed in a series of overlapping declination strips with their centres separated by 2°·75 and any source in the region surveyed was thus recorded above the half-power level. At each mast setting, recordings were made until a sufficient number of records had been accumulated to test the reliability of any particular feature.

The sensitivity of the equipment was checked each day by observations of the intense sources in Cygnus and Cassiopeia. A knowledge of the parameters of the equipment enabled an absolute calibration of the flux density scale to be made.

3. *The interferometer survey.*—This survey was carried out by a phase switched interferometer of the type described by Ryle (3) using the 218 ft paraboloid as one element and an array of dipoles, situated to the north-west of the paraboloid, as the second element. The beamwidth of this array between half-power points was 80° in declination and 4°·5 in right ascension. The zeros in the right ascension plane were arranged to fall on the first subsidiary maxima of the 218 ft paraboloid polar diagram and the side lobes of the resultant polar diagram in this plane were negligible. In the declination plane the resultant lobe envelope was effectively the voltage polar diagram of the paraboloid, and the first side lobes were 12 per cent of the peak intensity in the main beam.

The relative positions of the paraboloid and the dipole array were such as to give a lobe separation of 22′. There were about four lobes within the half-power points of the envelope of the resultant interference pattern which was approximately 3°·5 wide in right ascension and 4°·6 in declination.

Because of the large beam width of the dipole array in the north–south plane it was possible to survey different declination strips merely by tilting the mast of the 218 ft paraboloid. The interferometer survey was therefore carried out in the same way as the total power survey. Because of the freedom of the interferometer from interference it was possible to cover a given region of sky in a shorter time than that required in the total power survey and to reduce the spacing between successive declination sweeps to 1°·5. An absolute calibration of flux density scale was not attempted; the scale was related to the total power survey by reference to the prominent source I.A.U. 08N4A.

4. *Method of analysis.*—The records obtained with the total power equipment and the interferometer were analysed separately and two independent lists of sources obtained. For the purpose of this analysis a source on the total power records was defined as an increase in flux density corresponding to the aerial beam shape, and on the interferometer survey as a fringe pattern with an envelope

approximating to that of the interferometer response pattern. The total power list will include enhanced regions of emission of angular diameter less than or comparable to the aerial beam width. The interferometer list, however, will include only those sources with angular diameters much less than the lobe separation of 22′. On the definition of a source given above it is possible that many of the "sources" listed may not correspond to genuine discrete sources and this is to be understood subsequently when reference is made to sources.

4.1. *Total power analysis.*—In all about 200 total power records were obtained covering completely the region of sky between declinations 26°N and 70°N and the right ascensions 06^h to 19^h, and a smaller region outside these limits. The right ascension of any source was found from the time at which it was observed to transit the aerial beam. The declination was found by plotting the flux density observed at transit against beam elevation. This also gave the flux density of the source. As a check on the reliability of sources in this list a number of records obtained at 92 Mc/s were compared with the corresponding records obtained at 158 Mc/s in an earlier survey by Hanbury Brown and Hazard (4). Fig. 1 shows an example of records obtained at declination 52° 30′N. It can be

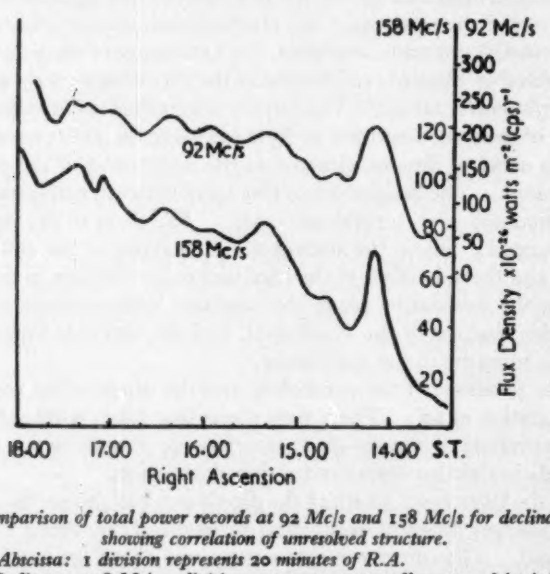

FIG. 1.—*A comparison of total power records at 92 Mc/s and 158 Mc/s for declination 52° 30′ N showing correlation of unresolved structure.*
Abscissa: 1 *division represents* 20 *minutes of R.A.*
Ordinate: 158 *Mc/s* 1 *division represents* 20×10^{-26} *watts* m^{-2} *(cps)*$^{-1}$.
92 *Mc/s* 1 *division represents* 50×10^{-26} *watts*$^{-2}$ *(cps)*$^{-1}$.

seen that although these records were taken at an interval of several years, there is very good agreement between them when allowance is made for the smoothing produced at 92 Mc/s by the larger aerial beam width. It was concluded from this comparison that the majority of the sources derived from the total power survey genuinely represent enhanced regions of intensity, although they may be due either to single sources or blends of two or more sources too close together to be resolved by the aerial beam.

4.2. *The interferometer analysis.*—The interferometer records were less disturbed by interference than the total power records and in general two records at

each declination setting were sufficient to establish the reliability of any particular feature. In all about 150 records were obtained, covering most of the region included in the total power survey. In general a continuous lobe pattern was visible above the noise and is probably due to faint sources below the resolving limits of the aerial. The interferometer was therefore not limited by noise fluctuations, but was resolution-limited.

In determining the right ascension of a source it was not possible to utilize the full accuracy possible with an interferometer because of the inclined base line and phase variations arising in the cable link between the two elements; the right ascension was therefore determined from the maximum of the envelope of the observed lobe pattern. The declination and flux density were found in the same way as in the total power survey, the flux density being related to the total power scale by reference to the source I.A.U. 08N4A as explained above.

The declination of each source was also estimated fom the lobe speed of the observed fringe pattern as a check that the source was not due to the presence of one of the intense sources in a minor lobe of the aerial beam. The main side lobes, which are most serious to the north and south of the main beam, are only important near to intense sources and, as their positions are known, they can be allowed for and so should not introduce any errors into the survey.

5. *Comparison of observations.*—The positions and flux densities of 116 sources were derived from the total power records and of 134 sources from the interferometer records. These lists of sources are to be published in the Jodrell Bank Annals together with a detailed comparison of the observations (5). The results of this comparison will now be summarized.

In the region of sky common to both surveys there are 81 sources in the total power list (list T) and 102 sources in the interferometer list (list I). A direct comparison of the lists shows that there are 40 positional agreements within the limits of experimental error. In addition one intense source on list T (Flux density $(S) = 160 \times 10^{-26}$ watts $m^{-2}(cps)^{-1}$) seems to be a blend of two intense sources on list I. On the basis of the estimated errors in position only 11 coincidences would be expected if the lists were completely random and therefore the majority of the above coincidences may be expected to be genuine. This conclusion is supported by a comparison of the flux densities of the sources in the two lists. In all cases there is no evidence of any deviation from a ratio of unity which cannot be accounted for by random errors in the measurements.

Table I

Comparison of total power (list T) and interferometer (list I) sources in the region common to both surveys

Flux density $\times 10^{-26}$ watts m^{-2} $(cps)^{-1}$	<20	20 to 39	40 to 59	>60
No. of list T sources	14	35	14	18
No. of list T sources not on list I	9	22	5	4
No. of list I sources	1	51	34	16
No. of list I sources not on list T	1	35	24	0

In the region of sky common to both lists there are 40 sources on list T and 60 sources on list I which do not coincide with sources in the other list. These discrepancies are divided into flux density groups in Table I.

It is to be noted that a source in a particular intensity range in one list may appear in a different range on the other list due to random errors in the flux

density measurements, and this accounts for apparent discrepancies between the source counts in this table. For example only 14 of the 18 list T sources with a flux density greater than 60×10^{-26} watts $m^{-2}(cps)^{-1}$ appear on list I, whereas 16 list I sources in this range coincide with list T sources.

Inspection of Table I shows that for sources of flux density less than 60×10^{-26} watts $m^{-2}(cps)^{-1}$ the agreement between the surveys is very poor. The discrepancies between the surveys cannot be due to systematic errors in position, which arise primarily because of uncertainties in the position of the beam of the 218 ft paraboloid, because the method of observation ensures that such errors are common to both surveys.

In order to investigate the discrepancies in more detail the two sets of records, which previously had been analysed quite independently, were compared directly. This direct comparison showed that there are sources on the interferometer survey in the position of a further five of the total power sources, while the total power records revealed four more sources which could be identified with interferometer sources. All of these sources were comparatively weak, the strongest

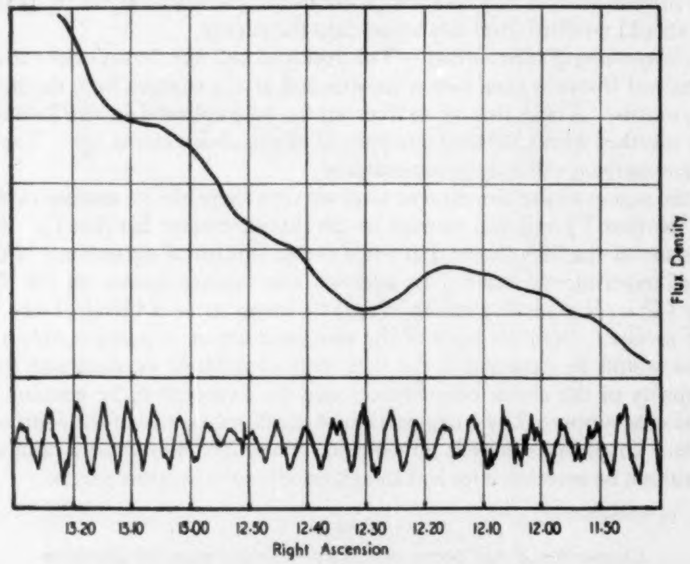

Fig. 2.—A comparison of the total power and interferometer records taken at declination 32° N. The flux density scales have been adjusted to be equal for both records.
Abscissa: 1 division represents 10 minutes of R.A.
Ordinates: 1 division represents 30×10^{-26} watts m^{-2} (cps)$^{-1}$.

having a flux density of 34×10^{-26} watts $m^{-2}(cps)^{-1}$, and they had been considered doubtful in the original analysis. In addition a number of total power sources were found to lie in very confused regions on the interferometer records where the fringe amplitude was compatible with the presence of the sources. Similarly a number of interferometer sources which lie close to sources on the total power list are in confused regions; the errors on either list may therefore be greater than estimated and some of these cases may in fact be coincidences. Other interferometer sources

lie in regions of enhanced intensity on the total power records where it is impossible to distinguish individual sources. There remain, however, ten total power sources near which there are definitely no sources of comparable flux density on the interferometer survey, and 34 interferometer sources which were definitely not observed on the total power survey. As the interferometer list contains only those sources whose angular diameter is less than 22′ it is possible that the sources observed on the total power records and not on the interferometer records have an angular diameter greater than this value.

The 34 sources observed on the interferometer records and not on the total power records present a more serious problem; although a source may appear on the total power but not on the interferometer survey because of its finite angular diameter, there is no analogous explanation to account for sources which are observed on the interferometer but not on the total power survey. It is possible that some of the weaker sources lie in depressions in the background radiation which mask their presence on the total power records, but it is improbable that this can account for all the discrepancies.

A comparison of a total power and interferometer record for declination N 32° is given in Fig. 2. The correlation between the records appears to be poor, and this is apparently due to effects of confusion.

6. *The reliability of a confusion-limited survey.*—It may be concluded from the results of the previous section that while some of the sources on the total power and not on the interferometer survey may have angular sizes greater than 22′, many of the cases where sources appear on one survey but not on the other are due to confusion. The confusion effects produce errors in both the total power and interferometer surveys, but it is to be expected that they are more serious in the interferometer survey because of its larger solid angle of reception. The intensity level at which discrepancies occur between the two surveys therefore enables an estimate to be made of the source density at which the confusion effects become important.

In the following analysis we have confined our attention to sources which were observed on the interferometer list but not on the total power list in order to avoid the effects of sources with large angular diameters. The analysis was therefore restricted to the region of sky between R.A. 05^h and R.A. 17^h. Outside this region the majority of the interferometer sources are strong sources found on isolated records and are therefore not a representative sample, but inside these limits the interferometer survey is complete. In this region there are 3500 sq. degrees of sky common to both surveys in which 96 interferometer sources were observed. Of these 96 sources there are 39 which have been identified with total power sources, and a further 22 which lie in confused regions and which might be present on the total power records, leaving 35 sources which were not observed. These sources are divided into flux density groups in Table II.

The figures given in column 4 refer only to those sources for which it could definitely be decided that no source of comparable flux density was present on the total power records.

The most important feature of the table is the dividing line which can be drawn at a flux density level of 60×10^{-26} watts $m^{-2}(cps)^{-1}$. All interferometer sources above this level were observed in the total power records but below this level there are serious discrepancies. The sources in the range 40–60 watts $m^{-2}(cps)^{-1}$ are of particular interest for it is very unlikely that sources of this flux density could

TABLE II

Counts of sources observed on the interferometer but not on the total power records

Flux density range ($\times 10^{-26}$ watts m^{-2} (cps)$^{-1}$)	Total no. of interferometer sources (list I)	Total no. of sources not listed on total power survey (list T)	No. of sources definitely not observed after a direct comparison of records
>60	11	0	0
51 to 60	13	9	5
41 to 50	20	14	7
31 to 40	18	14	10
31 to 30	33	19	12
<20	1	1	1

be present in the total power records and yet not be detected on a direct comparison of records. It therefore appears that at this level at least some of the sources in list I are spurious.

It may therefore be concluded that although the interferometer survey is reliable as regards the presence of sources above this flux density level, below this serious errors occur. Thus when the observed number of sources in an area of 3500 sq. degrees exceeds eleven, errors begin to appear in the interferometer survey. The area covered by the beam of the interferometer between half-power points is 12·5 sq. degrees and so this corresponds to a source density of about 1 per 25 beam areas. As these errors are probably caused by weaker sources in the beam this observed density corresponds to a higher density of confusing sources.

All the discrepancies given in columns 3 and 4 in Table II will not be due to errors in the interferometer survey but will also arise due to sources being overlooked in the total power survey. It is difficult to estimate the reliability limit of the total power survey because of the presence of large angular diameter sources, but as the solid angle of reception is less than that used in the interferometer survey it is to be expected that list T will be reliable down to a lower level than list I. This conclusion is supported by the figures given in Table I.

7. *Applications to other surveys.*—The type of error which arises due to confusion depends on the actual equipment used in the survey and the method of analysis adopted. However, the simple methods of analysis adopted here, which in the case of the interferometer survey neglected the phase information contained in the interferometer fringe pattern, are similar to those usually employed and it seems reasonable to assume that errors will begin to appear in other surveys at about the same observed density level. It is therefore of interest to apply this result to the results of other surveys. The two most important surveys so far published are the Cambridge 2C survey (**6**) and the Sydney survey (**7**). These will now be considered in turn.

7.1. *The Cambridge 2C survey.*—The Cambridge survey was carried out on 81·5 Mc/s using an interferometer technique and it lists 1936 sources. Shakeshaft *et al.* (**6**) have plotted the logarithm of flux density (S) against the logarithm of the total number of sources (N) above this flux density (log N/log S curve).

For a uniform distribution of sources the curve should have a slope of $-1{\cdot}5$; however, it shows a marked increase in slope above a flux density level of 70×10^{-26} watts m^{-2}(cps)$^{-1}$, and this has led Ryle and Scheuer (**8**) to suggest that the density of sources increases with distance, a suggestion which has profound cosmological implications.

The beam used in the Cambridge survey covers an area of sky of 11·8 sq. degrees. There will thus be one source per 25 beam widths at a source density of one per 295 sq. degrees, i.e. an observed density of 11 sources per steradian. The log N/log S curve for this survey shows that this source density occurs at a flux density level of 56×10^{-26} watts $m^{-2}(cps)^{-1}$. Below this level serious errors in the survey may be expected, the source counts cannot therefore be considered reliable. Inspection of the log N/log S curve shows that the increase in slope in this region is significant; moreover, although the majority of sources above this level are probably genuine, it is likely that the estimated flux densities near this level are considerably in error. Under these circumstances Bolton (**9**) has shown that an increase in slope compared with the true slope is to be expected. It is therefore concluded that the increase in slope observed in the Cambridge survey is based on source counts which must be considered unreliable, and it cannot be taken to have any cosmological significance.

7.2. *The Sydney survey.*—The aerial used in the Sydney survey was a "Mills Cross" 1500 ft long and, at the frequency of 85 Mc/s at which the survey was carried out, it has a beam area of 0·55 sq. degrees. This survey is not yet complete but a preliminary analysis has been carried out on 383 sources in a restricted region of the sky. The slope of the log N/log S curve deduced from these sources does not deviate significantly from $-1{\cdot}5$ down to a flux density level of 14×10^{-26} watts $m^{-2}(cps)^{-1}$; at this level the survey is sensitivity limited and the curve falls away. At the flux density level of 14×10^{-26} watts $m^{-2}(cps)^{-1}$ there are 200 sources per steradian which corresponds to an observed density of one per 25 beams. This is equal to the resolution limit derived above and so the survey should be free from serious errors due to confusion down to this level. As the slope of the log N/log S curve does not differ significantly from $-1{\cdot}5$, the Sydney survey therefore suggests that, at least for sources of flux density greater than 14×10^{-26} watts $m^{-2}(cps)^{-1}$, the spatial density does not increase with distance. This conclusion gives further support to the suggestion, made in the previous section, that the increase in slope of the Cambridge curve is not genuine but is due to inadequate resolution.

8. *Conclusions.*—It has sometimes been stated (**10, 11**) that the maximum number of sources which can be resolved in a radio survey is approximately equal to the number of beam areas in the sky. The observations presented in this paper show that in a practical case the number of sources which can be reliably catalogued is very much less than this. The actual density at which serious errors occur in a survey will depend on the instrument used and the method employed to reduce the observations; but it seems that if the simple type of analysis which is commonly employed in radio astronomy is used, then errors will occur at an observed density of about one source per 25 beam areas.

This limitation must be borne in mind when drawing conclusions from source counts and it places a severe restriction on the number of sources which can be reliably observed by existing aerials operating in the metre wavelength range where most surveys have been carried out.

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THE FLOW OF INFORMATION IN COSMOLOGICAL MODELS

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Summary

The rate of reception of information in a particular channel is defined as usual by $C = W \log (1 + b/N)$, where W is the band-width in which the signal power is b and the noise power N. W and b are expressed as functions of the Doppler ratio Z of the particular cosmological source under observation and are model-dependent. The steady-state model is chosen to illustrate this in detail, including a set of curves of $C(Z)$ which show, for instance, that $C(Z) \sim Z^{-4}$ for large Z. Results are given for other models and are discussed in relation to their visual horizons, if any.

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1. *Introduction.*—The idea of a horizon in cosmology was systematically treated by W. Rindler (1956), who defined a horizon as "a frontier between things observable and things not observable". He showed, for example, that in such model universes as the steady-state model there exists what he called an event-horizon: events taking place in certain distant systems after a particular epoch must remain forever unknown to a given observer. The present note undertakes to extend this sharp cut between observable and non-observable by studying the continuous gradation in the number of events that may be observed as the horizon is approached. Instead of considering the receipt of signals originating in specific events, we study rather the rate at which signals can flow to an observer from distant systems. In a typical case, as an observer using an arbitrary receiver for electromagnetic radiation looks further and further out into the universe, the information he can obtain from any given source system flows to him more and more slowly as the distance of that source increases. Finally it may reach the limiting zero rate. That limit marks an horizon if one exists. The present study exhibits the ways in which that limit is approached.

Just as the source systems are idealized in all models to a set of representative "particles" obeying the Weyl postulate, so the detailed events studied are idealized by the familiar statistical notions of information theory. We confine our attention to a single channel in each direction of observation, every channel possessing an assignable frequency pass band. This simulates a spectrograph or a radio-telescope well enough; the multi-channel properties of image-forming optical telescopes need an extension which we have not carried out in detail. The maximum information which any receiving channel can grasp varies as its distant object of study recedes. Three quantities determine this information flow rate: the channel noise, the frequencies handled, and the energy available from the source within the accepted frequency range. It is the influence of the cosmological line-element upon the third of these quantities which is the centre of interest.

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2. Rindler (1956) has shown that in the Robertson–Walker metric,

$$ds^2 = c^2\,dt^2 - R^2(t)\left\{\frac{dr^2 + r^2(\sin^2\theta\,d\phi^2 + d\theta^2)}{(1 + kr^2/4)^2}\right\}$$

which applies to all homogeneous and isotropic world models, the proper distance $l(t)$ from the origin O of a photon emitted at time $t = t_1$ by a source with the co-moving radial coordinate r is

$$l(t) = R(t)\{\sigma(r) - S(t_1, t)\} \tag{1}$$

where

$$\sigma(r) \equiv \int_0^r \frac{dr}{1 + kr^2/4} \quad \text{and} \quad S(t_1, t) \equiv \int_{t_1}^t \frac{c\,dt}{R(t)}. \tag{2}$$

$R(t)$ is the scale function of the metric and k the curvature index. The sources are the fundamental representative particles of the model, i.e. the galaxies. If the photon reaches O at time $t = t_0$,

$$\sigma(r) = S(t_1, t_0). \tag{3}$$

The Doppler ratio Z is defined by*

$$Z \equiv R(t_0)/R(t_1). \tag{4}$$

For any particular model equations (3) and (4) may be solved for r as a function of Z and t_0. For instance,

$$\begin{array}{llll} \text{if } R(t) = e^{t/T} & \text{and} & k = 0, & r = cTe^{-t_0/T}(Z - 1) \\ \text{if } R(t) = \alpha t^\alpha & (\alpha = 1), & k = 0, & r = ct_0^{1-\alpha}/a(\alpha - 1).(Z^{1-1/\alpha} - 1) \\ \text{if } R(t) = ct & \text{and} & k = -1, & r = 2(Z - 1)/(Z + 1). \end{array} \tag{5}$$

The first of these examples includes the steady-state model and the de Sitter model. The second includes the models of Page ($\alpha = 2$), Einstein–de Sitter ($\alpha = 2/3$), Dirac ($\alpha = 1/3$), etc. The third is Milne's model. We may therefore characterize a source by its Doppler ratio in a convenient and physically direct manner.

3. Let B be the absolute luminosity of the source (say in ergs/sec), b its apparent brightness (in ergs/cm²/sec), and L its luminosity distance (in cm). Then $b = B/4\pi L^2$ and

$$L = \frac{r}{1 + kr^2/4}\,\frac{R^2(t_0)}{R(t_1)}. \tag{6}$$

Further, let n be the number of sources per unit coordinate volume, so that the number between r and $r + dr$ is

$$N(r)\,dr = \frac{4\pi nr^2}{(1 + kr^2/4)^3}\,dr. \tag{7}$$

Since we are only dealing with isotropic models, 4π may be replaced by $d\Omega$ to give the number of sources in a shell subtending an angle $d\Omega$ at the origin. Now n is a constant in all models except the steady-state model. Leaving the latter for the moment, consider as an example the de Sitter model, which has the same metric. Using (5), (6) and (7) we have

$$b(Z) = \frac{B}{4\pi c^2T^2}\,\frac{1}{Z^2(Z - 1)^2} \tag{8}$$

and

$$N(Z)\,dZ = 4\pi nc^3T^3e^{-3t_0/T}(Z - 1)^2\,dZ \tag{9}$$

* This quantity is that usually designated in the literature by a small z. Note: $1 + z = Z$.

where $N(Z)\,dZ$ is now the number of sources with Doppler ratios between Z and $Z+dZ$. The total energy flux for all sources up to Z is

$$\int_1^Z b(Z)N(Z)\,dZ = BncT\,e^{-3t_0/T}(1-Z^{-1}). \tag{10}$$

The Z-dependencies of $b(Z)$ and $N(Z)$ for the other conservative models listed in (5) are shown in a table in the Appendix. The form of (10) is very similar for all these models: instead of Z^{-1} Page's model has $Z^{-3/2}$, the Einstein–de Sitter model has $Z^{-5/2}$, and Milne's model has Z^{-2}.

In the steady-state model the number of sources per unit coordinate volume, n, is not constant; instead, by definition of the steady-state, the number of sources per unit proper volume, m, is constant. Now the proper distance of a source at emission time $t=t_1$ is

$$l_s = R(t_1)\sigma(r) = r\,e^{t_1/T} = cT(Z-1)/Z$$

using (4) and (5), and the number of sources between l_s and l_s+dl_s is $4\pi l_s^2\,dl_s m$, so that for the steady-state model,

$$N(Z)\,dZ = 4\pi mc^3T^3\frac{(Z-1)^2}{Z^4}\,dZ \tag{9a}$$

and

$$\int_1^Z b(Z)N(Z)\,dZ = \frac{1}{5}BmcT(1-Z^{-5}). \tag{10a}$$

$b(Z)$ is of course the same as in (8) for the de Sitter model. (10a) shows that the maximum energy flux from all attainable sources is reached much more rapidly in the steady-state model than in the conservative models discussed above.

4. Let us now think of B as including only the part of the source power that is emitted in a well-defined frequency band of width W_1 and centred on a frequency ν_1, so that b is the corresponding energy flux received in a band of width W_0 and centred on frequency ν_0. The rate of reception of information (see, e.g., Bell (1956)) from this band W_1 is then

$$C(Z) = W_0 \log\,[1+b/N]$$

or

$$C(Z) = W_1 Z^{-1} \log\,[1+b(Z)/N] \tag{11}$$

where N is the noise power*. N includes local (say, atmospheric) background, stellar background, the thermal fluctuations in the information-carrying radiation itself, and the additional noise developed in the receiving instrument. In general N will be a function of the frequency range under consideration (i.e. the colour, the radio wave-length, or the gamma-ray energy) and of the receiving instrument, but for the purposes of the present investigation we shall be content to treat N as constant†. W_1 is fixed once the frequency range under observation has been decided upon. For instance, we may think of W_1 as the width of a single emission line common to a wide range of extra-galactic sources, and of $b(Z)$ as the intensity of the line. Or, if $b(Z)$ is to refer to the total apparent brightness of distant galaxies, then W_1 must be the suitably defined range of frequencies in which a galaxy emits the larger part of its radiation (such as the width at $1/e$ of the central maximum of a black-body spectrum). It is clear that for all expanding models

* The units of $C(Z)$ are bits per sec, hartleys per sec, or natural units per sec, according as the base of the logarithm is 2, 10 or e.

† For certain classes of instruments, for instance telescopes, a calculation of the information received would involve the *spatial* distribution of the energy flux over the receiver area as well as its *rate* of reception: each confusion circle of the image is really a distinct channel.

$(Z>1)$ the corresponding observed width W_0 is smaller and is centred on a lower frequency, since $W_0 = W_1/Z$ and $\nu_0 = \nu_1/Z$.

As an illustrative example we shall take the steady-state model again. Substituting (8) in (11) gives

$$C(Z) = W_1 Z^{-1} \log\left[1 + \frac{B}{4\pi c^2 T^2 N} \frac{1}{Z^2(Z-1)^2}\right] \tag{12}$$

and $C(Z) \rightarrow \text{const.} \times Z^{-5}$ as $Z \rightarrow \infty$. Fig. 1 shows several plots of equation (12) for different values of the parameter b_0/N where $b_0 = B/4\pi c^2 T^2$. The curves for other models are very similar. For instance, the Z-dependence of $C(Z)$ for large Z for the models of Page, Einstein–de Sitter, and Milne is respectively Z^{-4}, Z^{-3}, Z^{-5}. The curves illustrate clearly the very rapid reduction in the information rate with increasing Doppler ratio. They also show that a decrease in the noise power will produce a relatively much greater increase in the information rate from sources that already have high Doppler ratios than from those with low Doppler ratios.

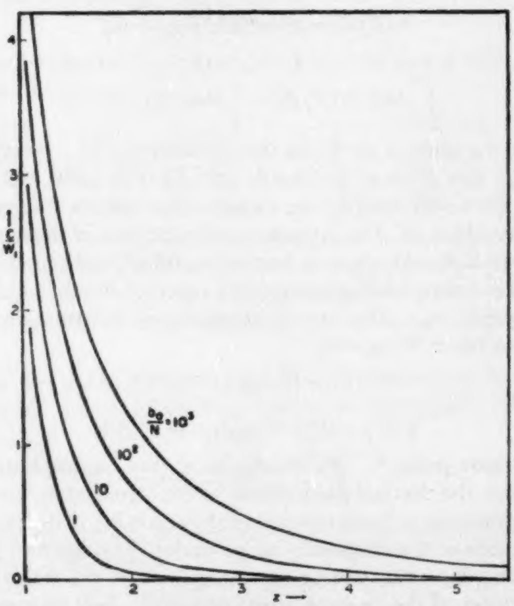

FIG. 1.—*The total rate of reception of information per unit source band width, C/W_1, as a function of the Doppler ratio Z of a source in the steady-state model, for four values of the parameter b_0/N, where $b_0 = B/4\pi c^2 T^2$ and N is the receiver noise power.*

$$\frac{C}{W_1} = \frac{1}{Z} \log\left[1 + \frac{b_0}{N} \frac{1}{Z^2(Z-1)^2}\right]. \tag{12}$$

5. As mentioned in the Introduction the steady-state model has an event horizon as defined by Rindler (1956). The necessary and sufficient condition for the existence of an event horizon is the existence of $S(t, \infty)$ in equation (1). Thus Page's model is another example. For such models it can be shown, using (3) and the definition of proper distance, that Z is inversely proportional to the proper distance, d say, of the source from the event horizon at the time of

emission of the signal. Thus as we consider a sequence of sources progressively closer to the event horizon, $d \to 0$, $Z \to \infty$, and $C(Z) \to 0$. Furthermore, sources actually *on* the event horizon at the time of emission can only be "seen" by the light they emitted when they were created ($t=0$), or, in the case of the steady-state model, by light emitted in the infinite past. However, this "light" has infinite wave-length and arrives with zero intensity, as equations (2) and (8) show. We are here only considering expanding models.

A second class of models, those for which $S(0, t)$ exists, have what Rindler called a particle horizon. Examples are the Einstein–de Sitter model and Dirac's model. Sources cross the particle horizon *toward* the observer (the event horizon is always crossed in a direction *away* from the observer) and first become visible by the light emitted at the time of creation, which again has infinite wave-length. Thus as we consider a sequence of sources with progressively larger Z, $C(Z) \to 0$ and we cannot obtain any information about the creation event.

From the point of view of the information rate, the difference between the event horizon and the particle horizon is that if we follow the time development of a single source near the former, its Doppler ratio increases and the rate at which we obtain information from it decreases, whereas the opposite is true for a source near the latter. However, in the case of a model with a particle horizon, Z for a particular source may increase again after some time. There are, in fact, models that have both types of horizon. On the other hand Milne's model has neither type of horizon.

6. The total rate of reception of information from all sources with Doppler ratios from one up to Z is

$$\int_1^Z C(Z)N(Z)\,dZ.$$

Since the integrals involved do not have simple analytic solutions, we shall consider instead the following integral,

$$\int_{Z_0}^Z C(Z)N(Z)\,dZ, \qquad Z_0 \geqslant 1 \tag{13}$$

which gives the total rate of reception of information from all sources with Doppler ratios between Z_0 and Z. Substituting (9*a*) and (12) in (13) and retaining only the first term in the expansion of the logarithm, we get for the steady-state model,

$$\int_{Z_0}^Z C(Z)N(Z)\,dZ = \frac{W_1 BmcT}{6N}\left(\frac{1}{Z_0^6} - \frac{1}{Z^6}\right), \qquad Z > Z_0 \geqslant 1. \tag{14}$$

The form of (14) is again very similar for the other models we have been considering except that instead of m the appropriate parameter is n for conservative models. The results are collected in the Table in the Appendix.

If we compare the expressions (11) and (13) for these various models it is evident both that the information rate from a single source falls off more rapidly with increasing Doppler ratio for the steady-state model than for the others (in Milne's model it falls off equally rapidly), and that the total information rate from all sources beyond a certain point ($Z = Z_0$) increases more rapidly up to its maximum value.

Finally we may note that as $Z \to \infty$ equation (14) becomes

$$\int_{Z_0}^\infty C(Z)N(Z)\,dZ = \frac{W_1 BmcT}{6N}\frac{1}{Z_0^6}, \qquad Z_0 \geqslant 1 \tag{15}$$

with similar expressions for the other models. (15) gives the total information rate from all accessible sources with $Z \geqslant Z_0$. This might be called "the residual information rate from beyond Doppler ratio Z_0". It decreases more rapidly with Z_0 for the steady-state model than for any of the others considered here. We see from (15) and from similar equations for the other models that the total information rate from all sources in a particular channel must be finite. This must be true for all expanding models.

7. As an example of a contracting model we shall briefly consider a model with $R(t)=e^{-t/T}$ and $k=0$. $b(Z)$ and $C(Z)$ are exactly as in (8) and (12), but now $0<Z<1$ instead of $1<Z<\infty$. Hence $b(Z)$ becomes infinite for very distant sources ($Z\to 0$), whereas for expanding models it becomes zero ($Z\to\infty$). Similarly it can be shown that the total energy flux, as in (10), increases to infinity as $Z\to 0$, whereas for expanding models it increases up to a finite value as $Z\to\infty$. This is to be expected, since $Z=0$ corresponds to photons of infinite frequency and energy. As for the information rate, it can be seen from (12) that as Z goes from one to zero the information rate from a single source, $C(Z)$, decreases rapidly at first to a minimum value and then increases again more slowly to infinity. What we have previously called the residual information rate from beyond Doppler ratio Z_0 is then, of course, infinite for all Z_0.

8. *Acknowledgments.*—We wish to express our thanks to Dr W. Rindler for his valuable and critical interest in this work.

APPENDIX 1

If B is the total absolute luminosity of the source, we may estimate it numerically. An average galaxy contains about 10^{10} stars, and an average star is approximately a black-body at 6000°K and of emitting area 10^{23} cm, giving $B=10^{44}$ ergs/sec. Hence with $T=13\times 10^9$ years (Sandage 1958) the parameter $B/4\pi c^2T^2$ in equation (8) for the steady-state model becomes approximately 10^{-13} ergs/cm²/sec.

We can also make a rough estimate of the noise power N for a particular instrument. For instance, the Palomar telescope can just distinguish a source of apparent magnitude $m\approx 23$, which corresponds approximately to $Z\approx 1{\cdot}5$, $v/c\approx 1/3$, and $b\approx 10^{-14}$ ergs/cm²/sec (since $m=-2{\cdot}5\log b-11{\cdot}6$). The noise power N must be of the same order of magnitude as this limiting apparent brightness, so that $N\approx 10^{-14}$ ergs/cm²/sec. Therefore in Fig. 1 the curve with $b_0/N=1$ should give a good indication for the Palomar telescope of the information rate (in a given region of the plate) as a function of the source Doppler ratio, provided that the de Sitter metric is approximately valid for the actual universe.

APPENDIX 2

Table I below summarizes the dependence on the Doppler ratio Z of the various quantities discussed in the text. A word ought to be said about columns 6 and 7. From col. 9 it is evident that $N(Z)$ and $\int_1^Z N(Z)\,dZ$ diverge for large Z for the models of Page, Milne and de Sitter, whereas for the other models $N(Z)$ goes to zero and $\int_1^Z N(Z)\,dZ$ asymptotically approaches a finite upper bound. For the models of Page and de Sitter, which incidentally both have an event horizon, it can be seen from the way in which equations (5) and (7) are used to

TABLE I

Model	$R(t)$	k	Type of horizon	Apparent brightness, $b(Z)$	No. of sources per unit range of Z, $N(Z)$	No. of sources per unit range of ν_0, $N(\nu_0)$	Limiting information rate, $C(Z)$, for $Z \geqslant 1$	$\int_{Z_0}^{\infty} C(Z)N(Z)\,dZ$ $Z_0 \geqslant 1$
Steady-state	$e^{t/T}$	0	event	$\frac{1}{Z^2(Z-1)^2}$	$\frac{(Z-1)^2}{Z^4}$	$-\frac{(\nu_1-\nu_0)^2}{\nu_1^2}$	Z^{-5}	Z_0^{-4}
de Sitter	$e^{t/T}$	0	event	$\frac{1}{Z^2(Z-1)^2}$	$(Z-1)^2$	$-\frac{(\nu_1-\nu_0)^2\nu_1}{\nu_0^4}$	Z^{-5}	Z_0^{-2}
Page	at^2	0	event	$\frac{1}{Z^2(Z^{1/2}-1)^2}$	$\frac{(Z^{1/2}-1)^2}{Z^{1/2}}$	$-\frac{(\nu_1^{1/2}-\nu_0^{1/2})^2\nu_1^{1/2}}{\nu_0^{5/2}}$	Z^{-4}	$Z_0^{-5/2}$
Milne	ct	-1	none	$\frac{1}{(Z^2-1)^2}$	$\frac{(Z^2-1)^2}{Z^3}$	$-\frac{(\nu_1^2-\nu_0^2)^2}{\nu_1^2\nu_0^3}$	Z^{-5}	Z_0^{-3}
Einstein–de Sitter	$at^{2/3}$	0	particle	$\frac{1}{Z(Z^{1/2}-1)^2}$	$\frac{(Z^{1/2}-1)^2}{Z^{5/2}}$	$-\frac{(\nu_1^{1/2}-\nu_0^{1/2})^2}{\nu_1^{3/2}\nu_0^{1/2}}$	Z^{-3}	$Z_0^{-7/2}$
Dirac	$at^{1/3}$	0	particle	$\frac{Z^3}{(Z^3-1)^2}$	$\frac{(Z^3-1)^2}{Z^7}$	$-\frac{(\nu_1^3-\nu_0^3)^2\nu_0}{\nu_1^8}$	Z^{-3}	Z_0^{-5}

get $N(Z)$ that the divergencies are due to coordinate distances $r(Z)$ which are monotonically increasing functions of Z. Conversely, $r(Z)$ is bounded in the Einstein–de Sitter and Dirac models, which both have a particle horizon. The intermediate case would be a mass-conserving model with $R(t)=ct$ and $k=0$, which would have neither type of horizon, and for which $N(Z)$ converges as $(\log Z)^2/Z$ whereas $\int_1^Z N(Z)\,dZ$ diverges as $(\log Z)^3$.

The divergence of $N(Z)$ and its integral in the de Sitter model leads to no difficulty if the model has a finite age, for then by (4) the maximum value of Z attainable is $Z=e^{t_0/T}$ (setting $t_1=0$), so that, even apart from considerations of information, $N(Z)$ and its integral are always bounded at a fixed reception time.

For Milne's model, equation (5) shows that $r(Z)$ is bounded. Here the source of the divergence of $N(Z)$ is the fact that the metric is hyperbolic ($k=-1$), which makes itself felt in the denominator of equation (7).

The steady-state model differs from all the others by not conserving mass. $N(Z)$ is calculated with the use of the proper distance of a source, $l_s=cT(Z-1)/Z$, which is bounded. In fact, the upper bound of l_s is cT, the constant proper distance of the event horizon from the origin observer, and

$$\int_1^\infty N(Z)\,dZ=(4\pi/3)mc^3T^3,$$

the constant total number of observable sources.

Col. 7 in the table illustrates another way of looking at the question of the number of sources. Equation (4) may be written $Z=\nu_1/\nu_0$. Suppose we fix our attention on a single emission line (say, the 21 cm hydrogen line), emitted by a *constant* fraction of the total number of sources*. Then ν_1 is fixed and ν_0 becomes smaller and smaller as we trace the line in the spectrum of sources with increasingly greater Z. We have $dZ=-\nu_1/\nu_0^2\,d\nu_0$, and $N(\nu_0)$ is defined by $N(Z)\,dZ=N(\nu_0)\,d\nu_0$. $N(\nu_0)\,d\nu_0$ then gives the number of sources with the line displaced from ν_1 to the region between ν_0 and $\nu_0+d\nu_0$ (apart from constant factors). Col. 7 shows that as $\nu_0\to 0$ $(Z\to\infty)$ $N(\nu_0)$ tends to a finite value for the steady-state model, to zero for Dirac's model, and diverges for the others. For the de Sitter model the divergence again goes only down to the minimum value of ν_0/ν_1, $e^{-t_0/T}$. Finally, the divergence of $N(\nu_0)$ for the Einstein–de Sitter model differs from that of Page's and Milne's model in that $\int_0^{\nu_1} N(\nu_0)\,d\nu_0$ must be finite for the former (since we have seen that $\int_1^\infty N(Z)\,dZ$ is finite for this model).

* In fact, the fraction would not remain constant for very large Z (except for the steady-state model), for the physical processes giving rise to the observed radiation must change drastically, as we approach the creation event ($Z\to\infty$), thus providing a cut-off for $N(\nu_0)$ as $\nu_0\to 0$ and preventing any divergencies.

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NUMBER COUNT RELATIONS IN OBSERVATIONAL COSMOLOGY

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Summary

Theoretical formulae relating the counts of sources of radiation to other observable quantities in cosmology are systematically presented in the form considered most relevant to present day problems. These relations are particularly directed to the practical detection of evolutionary effects in the galaxies, employing quantities directly measured by astronomers and taking account of the possible variation of these quantities with epoch. The analysis is based on the Robertson–Walker space-time metric for a "smoothed out" universe, which still seems likely to be the most fruitful method of approach at the present time.

The relations are designed to apply to the number counts of sources emitting in either the optical range of the spectrum or in the radio range, and these are determined in terms of both red shift and apparent magnitude. An alternative number count formula is derived which may possibly be more applicable in an evolutionary universe should radio sources arise by the collision of galaxies. Each relation is discussed in some detail with regard to the value of its application to the actual universe.

It is shown how the number counts may be employed to measure any secular rate of change that may exist in the strength of source emission, and how they may be utilized in conjunction with the red shift—apparent magnitude data to provide an independent measure of the extent of selection effect in the latter data. It is shown, also, how the number counts may independently provide the values of the Hubble parameter and the acceleration parameter of the expanding universe.

The paper concludes with a systematic comparison between the observable number count characteristics of an evolutionary universe and those of the steady-state model, indicating several means of distinguishing between these two types of universe.

1. *Introduction.*—Valuable as earlier work on observable relations in cosmology has been, much of it is inadequate to deal with the present programme of astronomers. This situation arises partly on account of new theoretical concepts concerning the universe, such as the steady-state theory, or the possibility of colliding galaxies giving rise to powerful radio sources. It also arises by reason of improvements in instrumental power and technique which make new or more refined types of observation possible.

The new science of radio astronomy, and counts of extragalactic radio sources in particular, as well as advances in technique associated with, for example, red shift—apparent magnitude correlations, multi-colour spectral analysis, measurements of angular distance between sources and clusters of sources and, it is to be hoped, more refined and accurate optical counts of galaxies, may all soon lead to the detection of significant evolutionary trends in the system of galaxies, or reveal the absence of any systematic effects of this nature.

* Received in original form 1959 March 9

It may therefore soon become possible by these means to resolve a fundamental issue of present day cosmology. This issue is whether it is appropriate to consider an evolutionary model of the universe as a whole, in which galaxies, once formed at approximately the same epoch, remain conserved in number, or whether we must regard the universe as being overall in a steady state for which the observed recession of the galaxies demands continual creation of matter and continual formation of new galaxies.

It is therefore necessary to derive formulae connecting astronomical observables which are particularly directed to the detection of evolution in the universe. In this paper observable relations involving the number counts of cosmic sources of radiation will be presented systematically in a form believed most suited to this end. Such counts have great theoretical importance and it is to be anticipated that improvements in instrumental power and technique will ultimately give them a reliable and authentic character. The red shift—apparent magnitude relation which was analysed in a manner consistent with the aims of the present paper in a previous publication (1), hereafter referred to as I, will also be reviewed summarily here. This is because the results of I are basically incorporated in the relations of this paper. The number count relations will in turn provide a means of determining independently the value of the evolutionary parameter occurring in the red shift—apparent magnitude relation, what proportion of this value is due to genuine evolution, and how much arises from an *apparent* evolution due to observational selection effects.

As in I we adopt, in common with earlier writers, the well known cosmological space-time metric

$$dS^2 = c^2dt^2 - R^2(t)\left\{\frac{dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2}{(1 + kr^2/4)^2}\right\} \tag{1.1}$$

derived by Robertson (2), and by Walker (3), for a universe that is spatially homogeneous and isotropic. That is, local geometrical effects arising from the mass concentrations occurring in the actual universe are neglected, so that this metric corresponds to an ideal universe in which matter is of uniform density at any given epoch t. This matter is supposed to move on the geodesics characterized by constant values of the spatial coordinates r, θ, ϕ. Since $R(t)$ is a disposable function of epoch t this ideal universe serves to represent the cosmic motions in the actual universe on a sufficiently large scale. The quantity $k/R^2(t)$ is the curvature of space at the epoch t, k being a constant which can take the values 1, 0, or -1.

It may be mentioned that the determinism associated with a "smoothed out" universe as described above has been criticized recently by Neyman and Scott (6) as being an unsatisfactory approach to the universe as it actually is, and these authors would recommend an indeterministic statistical analysis to take its place. Neyman and Scott (4), (5), (6) have established a statistical basis for the system of galaxies on the view that the universe is a realization of a stochastic process which is stationary in the three spatial coordinates (cosmological principle), and possibly also in time (perfect cosmological principle or steady state).

There is no doubt that systematic statistical studies of comparison and correlation among the galaxies would yield much valuable information. Such studies must in fact provide much of the data for the so-called deterministic approach adopted in this paper. Unfortunately, even Neyman and Scott find it necessary to assume a deterministic basis for the large scale motions of the galaxies (6).

Furthermore, their statistical theory is as yet derived only for Newtonian kinematics and Euclidean geometry, so that as a basis for determining the large scale character of space-time their analysis is at a great disadvantage compared with the relativistic treatment possible on the basis of the metric (1.1). For example, Neyman and Scott have to identify luminosity distance as found by astronomers with a Newtonian radial coordinate, leaving presumably for inclusion in an unknown error function the so-called "number effect" and "energy effect" on estimated distance which arise due to the expansion of space. Such effects can be dealt with quite naturally, however, in terms of relativistic kinematics.

Accordingly, until the statistical theory of Neyman and Scott has been extended to incorporate relativistic notions, a matter which promises to be formidable in its complication (cf. McVittie, 7), it seems likely that the deterministic basis adopted here will be the most fruitful in determining the large scale character of the universe which is the main concern in this paper. The deterministic observable parameters may of course be identified in practice with statistical mean values in the stochastic process envisaged by Neyman and Scott.

As in Paper I we transform the metric (1.1) to the more convenient form

$$dS^2 = c^2dt^2 - \frac{R^2(t)}{R_0^2}\left\{\frac{d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta\, d\phi^2}{(1+\rho^2/a^2)^2}\right\} \tag{1.2}$$

by means of the substitution

$$\rho = R_0 r, \tag{1.3}$$

where $R_0 \equiv R(t_0)$. We identify the epoch t_0 with the present epoch of observation of the universe, and regard it as fixed once and for all so that R_0 is a constant. We note that a^2 is related to the curvature at epoch t_0 by the equation

$$a^2 = 4R_0^2/k. \tag{1.4}$$

Our programme, in principle, is to relate the observable quantities employed by astronomers to the theoretical parameters $R(t)$ and a^2 occurring in (1.2), and then to develop relations between the observables which will involve these parameters as unknowns. By fitting these relations to the observational data it becomes possible to determine, in principle, those values of $R(t)$ and a^2 which best fit the large scale features of the actual universe. Since we shall allow for the possibility of systematic evolution in the characteristics of radiating sources, it will be possible also, given adequate data, to determine whether the universe is in fact evolutionary or in a steady state.

2. *Recapitulation of certain results of Paper I.*—It will be convenient for the development and understanding of the subsequent analysis to note here some results established in I:

(i) *Expansion of $R(t)$ and ρ in series.* The function $R(t)$ may be expanded by Taylor series in powers of $\tau(=t_0-t)$, the cosmic time lapse between emission of radiation from a distant source and its reception by the observer. Thus

$$R(t) = R_0(1 - \alpha_1\tau + \tfrac{1}{2}\alpha_2\tau^2 - \tfrac{1}{6}\alpha_3\tau^3 + \ldots.), \tag{2.1}$$

where $\alpha_1 = \dot{R}_0/R_0$, $\alpha_2 = \ddot{R}_0/R_0$, $\alpha_3 = \dddot{R}_0/R_0$, and so on, a dot indicating differentiation with respect to t and suffix o denoting quantities evaluated at the epoch of observation t_0. In this notation it was shown in I that the Taylor expansion of the radial coordinate ρ is

$$\rho = c\tau\left[1 + \tfrac{1}{2}\alpha_1\tau + \left\{\tfrac{1}{6}(2\alpha_1^2 - \alpha_2) + \frac{c^2}{3a^2}\right\}\tau^2 + \ldots.\right]. \tag{2.2}$$

(ii) ***Red shift* δ *and* δ–τ *relation.*** The Doppler effect on radiation from distant sources (of constant ρ, θ, ϕ coordinates) due to the expansion of the universe of metric (1.2), assuming $R(t)$ monotonic increasing, is such that

$$\frac{\lambda_0}{\lambda} = 1 + \delta = \frac{R(t_0)}{R(t)}. \tag{2.3}$$

Here λ is the wavelength of emission as measured at the source at epoch t, while λ_0 is the wavelength of the radiation when received at epoch t_0.

The expansion of the red shift parameter δ in powers of τ is

$$\delta = \alpha_1\tau + (\alpha_1^2 - \tfrac{1}{2}\alpha_2)\tau^2 + (\alpha_1^3 - \alpha_1\alpha_2 + \tfrac{1}{6}\alpha_3)\tau^3 + \ldots, \tag{2.4}$$

so that on inversion we derive

$$\tau = \left(\frac{1}{\alpha_1}\right)\delta + \left(\frac{\frac{1}{2}\alpha_2 - \alpha_1^2}{\alpha_1^3}\right)\delta^2 + \left(\frac{\alpha_1^4 - \alpha_1^2\alpha_2 + \frac{1}{2}\alpha_2^2 - \frac{1}{6}\alpha_1\alpha_3}{\alpha_1^5}\right)\delta^3 + \ldots, \tag{2.5}$$

a useful relation by which we can relate the red shift δ to other observables.

(iii) ***Heterochromatic apparent and absolute magnitudes.*** If l^* is the apparent luminosity of a distant source as registered on the detective device in the photographic or radio range, then, as shown in I, l^* is expressible in the form

$$l^* = \frac{\frac{1}{1+\delta}\int_0^\infty s(\lambda_0)\, E\left(\frac{\lambda_0}{1+\delta}, t\right) d\lambda_0}{4\pi D^2}. \tag{2.6}$$

Here

$$E\left(\frac{\lambda_0}{1+\delta}, t\right)\frac{d\lambda_0}{1+\delta}\ (= E(\lambda, t)\, d\lambda)$$

is the energy radiated per unit time in the emission waveband $d\lambda$ corresponding to the received waveband $d\lambda_0$, and as well as depending on wavelength λ it will possibly have an evolutionary dependence on the epoch t of emission which we have therefore allowed for. It should be noted, however, that such an evolutionary trend may be only *apparent* in practice, since it may result from a progressive effect of preferential selection by the observer of sources from the brighter end of the luminosity distribution. One means of distinguishing such an effect from genuine evolution is presented in Section 4.

The function $s(\lambda_0)$ of *received* wavelength takes account of the selective effects of the atmosphere and the sensitivity of the photographic plate or other detective device. Finally, the quantity D is the "luminosity distance" of the source, given in terms of the theoretical parameters of the metric (1.2) by the relation

$$D = \frac{\rho R_0}{(1 + \rho^2/a^2)\, R(t)}, \tag{2.7}$$

ρ being the (constant) space coordinate of the source.

Corresponding to (2.6) the registered *absolute* luminosity L_0^* of a standard source of the same general type in the neighbourhood of the observer will be

$$L_0^* = \frac{\int_0^\infty s(\lambda_0)\, E(\lambda_0, t_0)\, d\lambda_0}{4\pi 10^2}. \tag{2.8}$$

That is, we have put $D=10$ (parsecs) on the right-hand side of (2.6) in accordance with the conventional definition of absolute luminosity. In addition we have set $\delta=0$, $t=t_0$.

We now denote the registered heterochromatic apparent magnitude (i.e. photographic, photovisual, radio, etc.) of the distant source by m, and the corresponding registered absolute magnitude of the standard nearby source by M_0. Then m and M_0 will satisfy the relation

$$m-M_0=2\cdot 5\log_{10}(L_0{}^*/l^*). \tag{2.9}$$

(iv) *$m-\delta$ relation.* The important red shift—apparent magnitude relation in the form derived in I follows from the foregoing equations and is

$$m=5\log_{10}(c\delta)+M_0-5\log_{10}\alpha_1-5+1\cdot 086\left(1+\frac{\alpha_2}{\alpha_1{}^2}+\kappa^*+\lambda^*\right)\delta+O(\delta^2). \tag{2.10}$$

The coefficients κ^* and λ^* in the linear term are explained below.

(v) *Red shift "correction".* The quantity $1\cdot 086\kappa^*\delta$, when taken to the left-hand side of (2.10) is the orthodox correction, to the first order in δ, which used to be applied to the measured magnitude m to convert it to a bolometric magnitude in terms of which the theoretical relation was formerly written. However, it was shown in I that, in the presence of possible evolutionary effects on the emission spectrum, it is more direct to incorporate all "corrections" in a theoretical formula involving the magnitudes as registered and not to work in terms of bolometric magnitudes.

κ^* is given by the following equation

$$\kappa^*=1+\frac{\int_0^\infty s(\lambda_0)\lambda_0E'(\lambda_0,t_0)\,d\lambda_0}{\int_0^\infty s(\lambda_0)E(\lambda_0,t_0)\,d\lambda_0}, \tag{2.11}$$

where

$$E'(\lambda_0,t_0)=\frac{\partial}{\partial\lambda_0}\{E(\lambda_0,t_0)\}_{t_0}.$$

For a *narrow* waveband of reception $d\lambda_0$ in the neighbourhood of some effective wavelength λ_0, with $s(\lambda_0)=0$ outside this waveband, we can write approximately

$$\kappa^*=1+\frac{\lambda_0E'(\lambda_0,t_0)}{E(\lambda_0,t_0)}. \tag{2.12}$$

This would be a useful approximation in the case of colour photometry, or in radio reception, where the wavebands employed are narrow.

(vi) *Evolutionary "correction".* The term $1\cdot 086\lambda^*\delta$ in (2.10) arises from an assumed systematic evolution (or apparent evolution associated with selection effect) of the emission spectra of the sources being studied, and λ^* is given by the equation

$$\lambda^*=\frac{\int_0^\infty s(\lambda_0)\dot{E}(\lambda_0,t_0)\,d\lambda_0}{\alpha_1\int_0^\infty s(\lambda_0)E(\lambda_0,t_0)\,d\lambda_0}, \tag{2.13}$$

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The coefficients κ^* and λ^* in the linear term are explained below.

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For a *narrow* waveband of reception $d\lambda_0$ in the neighbourhood of some effective wavelength λ_0, with $s(\lambda_0) = 0$ outside this waveband, we can write approximately

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(vi) *Evolutionary "correction".* The term $1 \cdot 086\,\lambda^*\delta$ in (2.10) arises from an assumed systematic evolution (or apparent evolution associated with selection effect) of the emission spectra of the sources being studied, and λ^* is given by the equation

$$\lambda^* = \frac{\int_0^\infty s(\lambda_0)\dot{E}(\lambda_0, t_0)\, d\lambda_0}{\alpha_1 \int_0^\infty s(\lambda_0) E(\lambda_0, t_0)\, d\lambda_0}, \tag{2.13}$$

where

$$\dot{E}(\lambda_0, t_0) = \frac{\partial}{\partial t_0}\{E(\lambda_0, t_0)\}_{\lambda_0}.$$

For narrow wavebands this may be written approximately as

$$\lambda^* = \frac{\dot{E}(\lambda_0, t_0)}{\alpha_1 E(\lambda_0, t_0)}. \tag{2.14}$$

λ^* may also be related to the present rate of evolution of the corresponding heterochromatic absolute magnitude by the formula

$$\lambda^* = -0.92\, \dot{M}_0/\alpha_1, \tag{2.15}$$

the selectivity function $s(\lambda_0)$ remaining invariable in the time derivative of M_0.

(vii) *Information provided by the* $m-\delta$ *observational data.* The red shift—apparent magnitude relation (2.10) would provide, when fitted to the observational data, statistical values for the constants

$$M_0 - 5\log_{10}\alpha_1 - 5, \tag{2.16}$$

and

$$1.086\left(1 + \frac{\alpha_2}{\alpha_1^2} + \kappa^* + \lambda^*\right). \tag{2.17}$$

The parameter α_1 is Hubble's constant for the model in question, and a knowledge of the numerical value of (2.16) would yield this constant since the value of M_0 would be known from statistical studies of nearby sources of the specified type.

The expression (2.17) contains the important parameter α_2 which is positive or negative according as the expansion of the universe is accelerating or decelerating at the present epoch. Unfortunately, although κ^* can be derived by empirical calculation based on studies of local standard sources, we cannot arrive at even the sign of α_2 without a knowledge of the evolutionary parameter λ^*. Indeed, the value of this parameter must be regarded as one of the important quantities to be determined before the cosmological problem can be solved. One approach might be by estimation based on theories of evolution of stars and of observational selection effects, but much essential information is still required for such estimates to be relied on. However, λ^* may be derived independently by means of the number counts in conjunction with the $m-\delta$ data, as will be shown in this paper. The number counts may also provide independently the values of α_1 and α_2.

3. *Number counts related to red shift in an evolutionary universe.*—Formulae involving the numbers of nebulae in regions of space corresponding to given limits of apparent magnitude have been put forward by earlier writers, notably McCrea (8), Hubble and Tolman (9), Heckmann (10), McVittie (11, 12, 13), and Robertson (14). Our endeavour here will be to improve and add to their results according to the criteria stated in Section 1. It has been thought profitable to follow the presentation of each formula with a discussion of the procedure followed, the assumptions made, and the information that may be derived by application to the actual universe.

Preliminary to relating the number counts to other observables, we require the expression for the theoretical number of sources of a given character in the volume of space corresponding to limits $\rho = 0$ to general ρ of the cosmological metric (1.2) and observed from $\rho = 0$ at epoch t_0. This was obtained by McCrea (8) and is, in our notation,

$$N(\rho) = 4\pi n_0 \int_0^\rho \frac{\rho^2\, d\rho}{(1+\rho^2/a^2)^3}, \tag{3.1}$$

where n_0 is the number of sources in unit proper volume at epoch t_0.

By expansion of the integrand in (3.1) and integration term by term we find

$$N(\rho)=\frac{4}{3}\pi n_0\left\{\rho^3-\frac{9}{5}\frac{\rho^5}{a^2}+O(\rho^7)\right\} \tag{3.2}$$

so that, substituting for ρ in terms of τ by means of (2.2), we derive to sufficient approximation for our purpose

$$N(\tau)=\frac{4}{3}\pi n_0 c^3\left\{\tau^3+\frac{3\alpha_1}{2}\tau^4+O(\tau^5)\right\}. \tag{3.3}$$

Substitution now for τ in terms of δ by means of (2.5) yields the $N-\delta$ relation which it is convenient to write in the following logarithmic form

$$\log_{10} N(\delta)=3\log_{10}(c\delta)+\log_{10}\left(\frac{4}{3}\pi n_0/\alpha_1^3\right)+0{\cdot}651\left(\frac{\alpha_2}{\alpha_1^2}-1\right)\delta+O(\delta^2). \tag{3.4}$$

Discussion

(i) *Assumption of uniformity of source distribution.* Although the metric (1.2) corresponds to a universe that is spatially uniform, and the expression (3.1) for $N(\rho)$ refers to a uniform distribution of sources at each epoch t, contrary at least to the phenomenon of apparent clustering of galaxies, it may be assumed that the region of space dealt with is so large as to nullify any errors in this representation. The assumption expresses in effect the so-called "cosmological principle" according to which there is, on a sufficiently large scale, spatial isotropy in the distribution and motion of galaxies at each epoch t with respect to *every* "fundamental" observer who is at rest relative to the average motion of matter in his (sufficiently large) neighbourhood. It can be shown geometrically that this demands large scale spatial homogeneity. Our metric (1.2) represents such a universe with its local irregularities smoothed out.

All the observational evidence so far goes to justify the cosmological principle. This principle may be regarded as equivalent to the spatially stationary stochastic process envisaged by Neyman and Scott.

(ii) *Assumption of source conservation.* It must be pointed out that the formula (3.1) assumes number conservation of sources as the universe expands. In the case of ordinary galaxies this assumption may be justified by the argument that, in the absence of continual creation, the intergalaxial density of matter becomes so low due to the expansion that after a certain stage no further condensation into new galaxies is possible. On the other hand all galaxies once formed will be composed predominantly of stars which have a lifetime greater than the past period of expansion, as calculated approximately by the Hubble law, and so no galaxy could yet have ceased to radiate and become invisible.

However, the formula would not extend to those very distant regions in which, when the radiation was emitted, galaxies were still being formed. A rapid decrease of the source counts at such a range would nevertheless be direct evidence of an evolutionary universe. While this range is unlikely to be observed by the optical instruments it is possible that it may come into the radio range.

The formula (3.1) may not in any case apply to counts of radio sources in an evolutionary universe if these should arise by colliding galaxies, and an alternative number count formula which may be appropriate to such a phenomenon will be given in Section 5.

(iii) *Order of approximation in δ—the curvature of space.* The relation (3.4) neglects terms of order δ^2 and so does not take account of the curvature of space at the present epoch. Calculation shows that the coefficient of δ^2 contains the

curvature but is very complicated, including also terms such as α_3/α_1^3 and $(\alpha_2/\alpha_1^2)^2$. Thus it would not be profitable to analyse this term from the observational data, although in practice a term in δ^2 might be formally allowed for, to give correct weight to the coefficient of δ. It seems best at present to obtain the curvature for an evolutionary universe by appealing to the field equations of general relativity for a universe of metric (1.2), it being assumed that the parameters α_1 and α_2 have been already determined. The relevant analysis for this purpose has been given by Hoyle and Sandage (15) and will not be repeated here.

(iv) *The Hubble parameter* α_1. From the relation (3.4), fitted to the observational data, we may deduce a value for the quantity n_0/α_1^3. A knowledge of n_0, which would be obtained as the statistical average density of the sources at epoch t_0 (i.e. near the observer), would therefore permit a check on the value of the Hubble constant α_1 obtained otherwise from the $m-\delta$ relation (2.10).

(v) *The acceleration parameter* α_2/α_1^2. The importance of a relation between N and δ, such as is given by (3.4), seems to have been overlooked in the past with regard to its ability to provide directly the fundamental quantity α_2/α_1^2 from the coefficient of δ. The formula (3.4) assumes that the universe has an evolutionary character but a similar relation applicable to a steady-state universe will be given in Section 6.

It is to be noted that the uncertainty in deriving α_2/α_1^2 from the $m-\delta$ relation (2.10), because of the presence of the unknown evolutionary parameter λ^*, does not arise here. This is a great advantage which might justify the laborious measurements of red shift to the same extent as the counts for limited areas of the sky and a limited range of δ. It is very much to be hoped that advances in technique will ultimately permit the registration of red shifts on a sufficiently large scale for such a valuable programme to be carried out. It would be important in using this formula that δ should not be too large, for two reasons. First, the formula (3.1) assumes that *all* the sources in the region of space being studied can be observed, despite any dispersion in their absolute magnitudes. Secondly, (3.4) assumes that the red shift of every source counted can be obtained. On the other hand the range would not have to be too small, since otherwise errors arising from the effect of clustering would not be averaged out.

(vi) *The evolutionary parameter* λ^*. The value of α_2/α_1^2 obtained as in (v) would, by substitution into the expression (2.17) obtained from the $m-\delta$ relation, permit the determination of the evolutionary parameter λ^* for an evolving universe. However, care must be exercised before attributing this value of λ^* entirely to genuine overall evolution, since it may partly arise as an effect of selection. A means of deriving that part of λ^* due to genuine evolution, other than by the theory of evolution of stars, is given by the $m-\log N$ relation analysed in Section 4.

4. *Number counts related to apparent magnitude in an evolutionary universe.*—By eliminating δ between the equations (2.10) and (3.4) we find, neglecting terms of order δ^2,

$$\log_{10} N(m) = \log_{10}(4000\pi n_0/3) + 0{\cdot}6(m - M_0) - 6{\cdot}51(2+\kappa^*+\lambda^*)\left(\frac{\alpha_1}{c}\right) 10^{0{\cdot}2(m-M_0)}. \quad (4.1)$$

Thus $N(m)$ represents the number of sources of apparent magnitude less than or equal to m. As in (2.10) the parameter M_0 is the heterochromatic absolute magnitude of a standard source at the epoch t_0 of observation.

On differentiating this relation we find for its gradient, neglecting δ^2,

$$\frac{d(\log_{10} N)}{dm} = 0{\cdot}6\left\{1 - 5(2+\kappa^*+\lambda^*)\left(\frac{\alpha_1}{c}\right) 10^{0{\cdot}2(m-M_0)}\right\} \tag{4.2}$$

or, in terms of δ,

$$\frac{d(\log_{10} N)}{dm} = 0{\cdot}6\{1 - \tfrac{1}{2}(2+\kappa^*+\lambda^*)\delta + O(\delta^2)\}. \tag{4.3}$$

Discussion

(i) *The parameter* λ^*. From equations (4.1) or (4.2) fitted to the number count data we should be able to derive a value for $\kappa^* + \lambda^*$ (the Hubble parameter α_1 and the standard absolute magnitude M_0 being assumed known). Since κ^* is a quantity which may be derived empirically by examination of the spectrum of the standard source, we thus arrive at a value for λ^* appropriate to the evolving universe that we are considering.

Now, in using the $m - \log N$ relation it is the apparent magnitude of the source that is measured and not its red shift. Thus, although we obtained this formula by relating δ to m in accordance with (2.10), it is appropriate here to attribute λ^* to genuine evolution only, and not partly to a selection effect that might arise with an actual measure of red shift. This is because of the very much greater sample of galaxies that can be examined in the case of the $m - N$ observations, thus reducing the selection effect. In fact our formula (3.1) for N assumes that *all* the sources in any region of space can be observed, and indeed it must be remembered that it may be possible, if the distance be not too great, to determine the apparent magnitude of all sources in a given region whereas it might not be possible to determine all their red shifts. Naturally, however, there must be an upper limit, depending on the luminosity distribution of the sources and the sensitivity of the photographic plate, etc., to the values of m employed in the data if the formulae are to be correctly applied.

(ii) *Selection effect in the red shift—apparent magnitude data.* The determination of λ^* by the method indicated in Section 3 (vi) minus that value for λ^* determined as in (i) above should indicate, for the case of an evolutionary universe to which the present number count relations apply, how much of the evolutionary parameter occurring in (2.10) is due to a selection effect in the choice of sources for the red shift measurement. This would determine the first time derivative of M_0 arising from selection effects and would provide valuable information regarding the range of the luminosity distribution for not too distant sources. However, it must be pointed out that the second time derivative of M_0, arising from both evolutionary changes and selection effects, might be of significant order for $m - \delta$ measurements pursued to greater distances than are contemplated for the $m - N$ relation. The second time derivative of M_0 would come into the terms of higher order which are not evaluated explicitly in our formulae.

Other means of detecting the presence of $m - \delta$ selection effect, based on measurements of angular distribution, will be dealt with in a later paper.

(iii) *Measures of evolution* (*optical counts*). The value of λ^* determined as in (i) above will indicate, in accordance with (2.13) and (2.15), the present rate of genuine evolution in the intrinsic luminosity of a typical source.

We see from (4.2) that the gradient of the $m - \log N$ curve for $\delta \simeq 0$ is equal to 0·6. A gradient steeper than this associated with more remote regions of space could arise in the next approximation only if $\kappa^* + \lambda^* < -2$. For photographic

magnitudes the tables given by Humason, Mayall, and Sandage in their paper presenting the red shift data (**16**) indicate that the orthodox red shift "correction", i.e. neglecting evolutionary effects, is to good approximation a linear one corresponding in our formalism to $\kappa^* \simeq 4{\cdot}3$. Consequently, a gradient steeper than 0·6 in the photographic wavelengths would require $\lambda^* < -6{\cdot}3$. By (2.15) this means $\dot{M}_0 > 6{\cdot}8\alpha_1$. Taking $1/\alpha_1 < 2 \times 10^{10}$ yrs, we derive $\dot{M}_0 > 0{\cdot}34$ mag/10^9 yrs. Such a rapid rate of evolution required for a gradient steeper than 0·6 makes it unlikely that such an effect will be found in the photographic range.

(iv) *Measures of evolution (radio counts).* At radio wavelengths a gradient steeper than 0·6 has been found for the extragalactic radio sources by Ryle and his co-workers at Cambridge (**17**, **18**). (These authors graph $\log N$ against $\log I$ where I is the flux of energy per unit frequency of the fixed waveband of reception used. Consequently, the value of a gradient which is 0·6 in our notation would be $0{\cdot}6 \times -2{\cdot}5 = -1{\cdot}5$ in theirs and they actually find a gradient steeper than -2.) We shall assume in the present discussion that radio sources are distributed uniformly in space at each epoch t. If in addition their density in space is a *constant* fraction of that of ordinary galaxies, whether they arise by the collision of galaxies or not, then formulae (4·1) and (4.2) would still be valid for ***sufficiently near*** sources with m, M_0, κ^* and λ^* now referring to radio wavelengths.

The radio spectrum appears to obey a power law so that the flux density $I \propto \nu^x$ where x is a constant and ν is the frequency. The average value of x for the nearer extragalactic sources has been found to be $-1{\cdot}2$ (**19**). Accordingly, in the notation of Section 2, the rate of emission per unit ***wavelength*** at the present epoch will be $E(\lambda, t_0) \propto \lambda^{-0{\cdot}8}$. Whence, referring to (2.12) we find that for radio wavelengths in narrow wavebands $\kappa^* = 0{\cdot}2$ on the average. Consequently, since a monotonic increase of the gradient of the $m - \log N$ curve from the local value of 0·6 must theoretically show itself in the coefficient of the term of order δ, this requires $\lambda^* < -2{\cdot}2$. By (2.15), taking once again $1/\alpha_1 < 2 \times 10^{10}$ yrs, this means that the standard absolute radio magnitude M_0, in the appropriate range of wavelength, must be increasing at the present epoch at a rate equal to 0·1 mag/10^9 yrs at the minimum. The required rate will exceed this value in proportion as $1/\alpha_1$ is less than 2×10^{10} yrs. In any event it is clear that considerably less rapid evolutionary changes are required in the radio case than in the optical.

Although it does not affect the validity of the above theoretical considerations for an ideally smooth distribution of sources a possibly serious objection to the application of our formulae to the radio counts may be raised on the grounds that, as Ryle believes, the radio sources that have been detected are so sparsely distributed that the majority of them are at very great distances ($\delta \simeq 1$). Thus limited expansion in series, even to order δ^2, may be of doubtful validity at this range. Without knowledge of the very cosmological solution that we are seeking we cannot say how rapidly the Taylor series would converge at $\delta \simeq 1$. There is at least the fortunate circumstance that the radio spectrum appears to have no kinks in it which would seriously affect red shift approximations at this range. Nevertheless, a more suitable procedure for radio counts might be to extend (4.1), without explicitly calculating the higher coefficients, in the form

$$\log_{10} N(m) = \log_{10}(4000\pi n_0/3) + 0{\cdot}6(m - M_0) - 6{\cdot}51(2 + \kappa^* + \lambda^*)\left(\frac{\alpha_1}{c}\right) 10^{0{\cdot}2(m-M_0)} + A\,10^{0{\cdot}4(m-M_0)} + B\,10^{0{\cdot}6(m-M_0)}, \tag{4·4}$$

with a similar extension of (4.2) yielding

$$\frac{d(\log_{10} N)}{dm} = 0{\cdot}6 \left\{ 1 - 5(2 + \kappa^* + \lambda^*) \left(\frac{\alpha_1}{c}\right) 10^{0{\cdot}2(m - M_0)} + 1{\cdot}54 A 10^{0{\cdot}4(m - M_0)} + 2{\cdot}30 B 10^{0{\cdot}6(m - M_0)} \right\}. \tag{4.5}$$

A few of the most powerful radio sources have been identified optically as colliding galaxies, and it has been suggested that the majority of the more distant sources are in fact of this nature. It will be of interest to establish in Section 5 an alternative number count formula which may in certain circumstances be more appropriate to this possibility in an evolutionary universe.

5. *An alternative number count formula for radio sources in an evolutionary universe.*—If radio sources arise by a kinetic effect then they may not always represent, independently of epoch, a constant fraction of the population of ordinary galaxies which the *Discussion* (iv) of Section 4 assumes. If collisions occur mainly inside clusters, and if these are assumed not to share in the expansion of the universe then, taken statistically, the radio sources might indeed represent a constant fraction of ordinary galaxies. But it may be that the formation of clusters is a comparatively recent process in an evolutionary universe so that these considerations may not extend to earlier epochs when the universe was in a more contracted state.

Accordingly, as an illustration of the possibilities we shall derive a formula applicable to an expanding universe in which radio sources arise by the 'molecular' collisions of galaxies uniformly distributed, i.e. without clustering, at each epoch t.

If $n(t)$ is the number density of galaxies at epoch t then, by reason of assumed conservation of galaxies, we can write

$$n(t) R^3(t) = n_0 R_0^3, \tag{5.1}$$

where as before n_0 is the value of $n(t)$ at the epoch t_0 of observation.

Let $\eta(t)$ be the number density of colliding galaxies. Then we may put

$$\eta(t) = \beta n^2(t) \quad (\beta \text{ constant}). \tag{5.2}$$

This may be justified as follows. If $a(t)$ is the cross-section of the average galaxy for collisions then a galaxy will make approximately $n(t)\ a(t)\ V(t)$ collisions in unit time, $V(t)$ being the average random velocity of a galaxy relative to the background stratum of galaxies. Each collision will last a time of order $a^{1/2}/V$ so that the number of collisions occurring in unit volume at any time will be approximately $n^2 a^{3/2}$. We now make the assumption that the variation of a with t can be neglected for our purposes and so (5.2) follows.

In terms of the metric (1.2), therefore, the number $N(\rho)$ of colliding galaxies whose radial coordinate is less than or equal to the general value ρ will be

$$N(\rho) = 4\pi \int_0^\rho \eta(t) \cdot \frac{R^3(t)}{R_0^3} \cdot \frac{\rho^2\, d\rho}{(1 + \rho^2/a^2)^3}. \tag{5.3}$$

Here t, the epoch of emission of radiation from the source of coordinate ρ, is a function of ρ in accordance with the equation of the null geodesic world line of the radiation, namely

$$cR_0 \int_t^{t_0} \frac{dt}{R(t)} = \int_0^\rho \frac{d\rho}{1 + \rho^2/a^2}. \tag{5.4}$$

Hence by (5.1), (5.2), and (5.3)

$$N(\rho)=4\pi\eta_0 R_0^3\int_0^\rho \frac{\rho^2\,d\rho}{R^3(t)(1+\rho^2/a^2)^3}, \tag{5.5}$$

where $\eta_0=\beta n_0^2$ is the density of sources arising by collision of galaxies at the present epoch.

Using now (2.1), (2.2), and (2.5) we may straightforwardly derive an expansion for $\log N$ in terms of δ analogous to (3.4). Elimination of δ by means of (2.10) finally yields the $m-\log N$ relation analogous to, and of the same order of approximation as, (4.4):

$$\log_{10} N(m)=\log_{10}\left(\frac{4000\pi\eta_0}{3}\right)+0{\cdot}6(m-M_0)-6{\cdot}51(\tfrac{1}{2}+\kappa^*+\lambda^*)\left(\frac{\alpha_1}{c}\right)10^{0\cdot2(m-M_0)}$$
$$+A'\,10^{0\cdot4(m-M_0)}+B'\,10^{0\cdot6(m-M_0)}, \tag{5.6}$$

A' and B' being constant coefficients not specifically evaluated. For the gradient of this relation we then derive

$$\frac{d(\log_{10}N)}{dm}=0{\cdot}6\left\{1-5(\tfrac{1}{2}+\kappa^*+\lambda^*)\left(\frac{\alpha_1}{c}\right)10^{0\cdot2(m-M_0)}\right.$$
$$\left.+1{\cdot}54A'\,10^{0\cdot4(m-M_0)}+2{\cdot}30B'\,10^{0\cdot6(m-M_0)}\right\}. \tag{5.7}$$

Discussion

According to (5.7) a monotonic increase in the gradient of the $m-\log N$ curve would require $\lambda^*<-0{\cdot}7$, if as before we take $\kappa^*\simeq0{\cdot}2$ for the radio spectrum. By (2.15), therefore, with $1/\alpha_1<2\times10^{10}$ yrs this corresponds to $\dot{M}_0>0{\cdot}04$ mag/10^9 yrs. This is a substantially smaller minimum rate of evolution for the present epoch than that found in Section 4 (iv) for the same gradient phenomenon.

In an expanding universe we can expect the average random velocity $V(t)$ of galaxies to have been greater in the past. Accordingly, since the collisions between galaxies would then have been more intense, and the galaxies themselves would have had more gaseous content, an evolutionary rate of the amount specified is perhaps plausible.

We emphasize, however, that the present discussion of an ideal universe devoid of clusters is for illustration only of the possible effects of conditions in the remote regions of the actual universe. We do not imply that the first order terms in (5.6) or (5.7) necessarily reflect the conditions of the actual universe in our neighbourhood.

6. *Number count relations in a steady-state universe.*

(i) $\delta-\log N$ *relation.* As for the evolving models we shall assume exact uniformity of distribution of the sources at any epoch t, thus neglecting statistical variations on a scale smaller than the range of the counts. In addition, in a steady-state universe the number n of sources in unit proper volume must remain constant independent of epoch. Since the steady-state metric is got from (1.2) by putting $R(t)=e^{t/T}$ (T constant) and $a^2=\infty$ (22, 23) we obtain, therefore, instead of (3.1) the following expression for $N(\rho)$:

$$N(\rho)=4\pi n e^{-3t_0/T}\int_0^\rho e^{3t/T}\rho^2\,d\rho \tag{6.1}$$

$$=4\pi n\int_0^\rho e^{-3\tau/T}\rho^2\,d\rho \tag{6.2}$$

where, as before, $\tau=t_0-t$.

We can expand N in series employing the relations (2.2) and (2.5) for the special case of the steady state. Putting $\alpha_1 = 1/T$, $\alpha_2 = 1/T^2$, we therefore derive for the $\delta - \log N$ relation

$$\log_{10} N(\delta) = 3 \log_{10}(c\delta) + \log_{10}\left(\frac{4}{3}\pi n T^3\right) - 0.977\delta + O(\delta^2). \tag{6.3}$$

(ii) *$m - \log N$ relation.* For case of the steady-state model we can put $\lambda^* = 0$ as far as the number count relations are concerned (see *Discussion* (ii)). Accordingly, the elimination of δ between (6.3) and (2.10) yields to the order of our approximation

$$\log_{10} N(m) = \log_{10}\left(\frac{4000\pi n}{3}\right) + 0.6(m - M_0) - 6.51\left(\frac{7}{2} + \kappa^*\right)\left(\frac{1}{cT}\right) 10^{0.2(m-M_0)}. \tag{6.4}$$

To the same order the gradient of this relation is

$$\frac{d(\log_{10} N)}{dm} = 0.6\left\{1 - 5\left(\frac{7}{2} + \kappa^*\right)\left(\frac{1}{cT}\right) 10^{0.2(m-M_0)}\right\}, \tag{6.5}$$

or, in terms of δ,

$$\frac{d(\log_{10} N)}{dm} = 0.6\left\{1 - \tfrac{1}{2}\left(\frac{7}{2} + \kappa^*\right)\delta + O(\delta^2)\right\}. \tag{6.6}$$

(iii) *$m - \log N$ relation (radio counts).* It is in the nature of the steady-state model that the statistical value of the density of radio sources will be, at every epoch and location, a constant fraction of the statistical density of ordinary galaxies, whether they arise by the collision of galaxies or not. Therefore, our $m - \log N$ formula for radio counts will simply be an appropriate extension of (6.4) to include higher order terms, of the form

$$\log_{10} N(m) = \log_{10}\left(\frac{4000\pi n}{3}\right) + 0.6(m - M_0) - 6.51\left(\frac{7}{2} + \kappa^*\right)\left(\frac{1}{cT}\right) 10^{0.2(m-M_0)} + F\,10^{0.4(m-M_0)} + G\,10^{0.6(m-M_0)}. \tag{6.7}$$

The gradient of the $m - \log N$ curve is then

$$\frac{d(\log_{10} N)}{dm} = 0.6\left\{1 - 5\left(\frac{7}{2} + \kappa^*\right)\left(\frac{1}{cT}\right) 10^{0.2(m-M_0)} + 1.54\,F\,10^{0.4(m-M_0)} + 2.30\,G\,10^{0.6(m-M_0)}\right\}. \tag{6.8}$$

(iv) *Exact relations for the steady-state model.* Because of the well defined nature of the steady-state model it is possible to derive exact expressions for the number count relations in terms of δ as a parameter. This will be an advantage in observational programmes that directly test for a steady-state universe.

By (5.4) we can write for the coordinate ρ in a steady-state universe

$$\rho = cT\,(e^{\tau/T} - 1), \tag{6.9}$$

and by (2.3) we can write τ in terms of δ, thus:

$$e^{\tau/T} = 1 + \delta. \tag{6.10}$$

Substitution from (6.9) and (6.10) into (6.2) then yields for $N(\delta)$ the expression

$$N(\delta) = 4\pi n c^3 T^3\left\{0.434 \log_{10}(1 + \delta) - \frac{\delta(2 + 3\delta)}{2(1 + \delta)^2}\right\}, \tag{6.11}$$

a result originally given by Bondi and Gold (22).

Turning to the $m-\log N$ relation an exact expression for the heterochromatic apparent magnitude m of a source in terms of δ may be derived from the summarized analysis of Section 2 or the more detailed work of Paper I. This may be written, for the case when there is no evolution of the source,

$$m = 5\log_{10}\{cT\delta(1+\delta)\} + K_0(\delta) + M_0 - 5, \tag{6.12}$$

where K_0 is the exact red shift "correction" defined by equation (4.8) of Paper I. In the linearised approximation of Section 2, $K_0 = 1{\cdot}086\,\kappa^*\delta$. As in (ii) above the evolutionary correction is not included here for the reason stated in *Discussion* (ii) below. If we now differentiate (6.11) and (6.12) we obtain the exact expression for the gradient of the $m-\log N$ curve in terms of δ:

$$\frac{d(\log_{10}N)}{dm} = \frac{2\delta^3}{\left\{5(1+2\delta)+2{\cdot}303\delta(1+\delta)\dfrac{dK_0}{d\delta}\right\}\left\{4{\cdot}606(1+\delta)^2\log_{10}(1+\delta)-\delta(2+3\delta)\right\}}. \tag{6.13}$$

Discussion

(i) *The Hubble parameter.* In the case of a steady-state universe the Hubble constant α_1 is equal to $1/T$. Thus, as in the case of the evolutionary models, the $\delta-\log N$ data would provide a check on the value of this constant, obtained otherwise from the $m-\delta$ relation, by substituting in the present case the value of n derived from local observations into (6.3).

(ii) *The parameter λ^*.* In the number count relations (6.4), (6.5), (6.6), we have put $\lambda^*=0$, and neglected evolution in (6.12) and (6.13). This is because in the case of the steady-state model a non-zero value of λ^* in the $m-\delta$ relation (2.10) could arise only in virtue of an *apparent* evolutionary trend in the observed spectra due to selection effect (cf. Paper I). Accordingly, as mentioned in the case of the evolutionary models in Section 4 (i), since the envisaged application of the number count relations implies absence of selection effects, it is appropriate before eliminating δ in (6.3) by means of (2.10) to put $\lambda^*=0$. The number count relations for the steady-state model, if the conditions for their validity are fulfilled, are therefore particularly easy to check against the observed relations.

(iii) *Selection effect in the $m-\delta$ data in a steady-state universe.* Assuming that the steady-state model had been established by means of the number count data (cf. Section 7), we should then be able to substitute $\alpha_2/\alpha_1^2=1$ in the $m-\delta$ relation (2.10). Accordingly, a knowledge of κ^* derived from the standard local spectrum would allow us to deduce the value of λ^* relevant to the red shift data in a steady-state universe. As mentioned in (ii) above this value of λ^* must arise solely from selection effect in this case. Consequently, the knowledge of the value of λ^* so derived would be valuable since it would provide some indication of the range of the luminosity distribution of sources over a limited region of a steady-state universe. Of course, the luminosity distribution could only be fully assessed in a steady-state universe by taking into account the most widely scattered observable sources of the same age, since among these would be the oldest observable sources and therefore the most advanced in individual evolution.

7. *The number counts as criteria for distinction between an evolutionary universe and a steady-state universe.* We recall here that, in any comparison between the

theoretical relations presented in this paper and those relations actually observed, terms of higher order may be formally allowed for in the process of curve fitting by least squares although they have been omitted here on the grounds that they are too complicated for practical analysis. This would where necessary give greater accuracy to the important coefficients determined for the terms of lower order.

(i) $\delta - \log N$ *relation.* The formula (6.3) provides a definite coefficient of δ, approximately equal to -1, which would have to be satisfied by the $\delta - \log N$ relation in a steady-state universe. Comparing with the corresponding relation (3.4) for the optical counts in an evolutionary universe we see that for the same value of α_2/α_1^2 as applies for the steady-state model, namely unity, the coefficient would be zero. This is one aspect of the great value of the number count relations in distinguishing between these two kinds of universe.

If, therefore, a coefficient of δ substantially different from -1 were found from the $\delta - \log N$ data the steady-state model would be ruled out. On the other hand the steady-state model could not be definitely *established* on the basis of this relation alone since the same coefficient of δ would be found in an evolutionary universe for which $\alpha_2/\alpha_1^2 = -\frac{1}{2}$. Decisive distinction in this case could nevertheless be made by appealing to the exact relation for the steady-state model given by (6.11), or alternatively to the $m - \log N$ data.

(ii) $m - \log N$ *relation (optical counts).* Since κ^* is known the relations (6.4), (6.5), (6.6) provide definite coefficients in the respective terms of order δ for the steady-state model. It will be sufficient for illustration to consider the coefficient of δ in (6.6). This is $-0{\cdot}3(\frac{3}{2} + \kappa^*)$. In the corresponding formula (4.3) for evolutionary models the coefficient is $-0{\cdot}3(2 + \kappa^* + \lambda^*)$. Since it is plausible to assume that the parameter λ^* will be negative for an evolving universe in which there is no creation of matter, it follows that the coefficient in this case will be definitely greater than for the steady-state model. Although κ^* is relatively large for the photographic range, taking a value in the neighbourhood of 4 (Section 4 (iii)), the detection of this difference in the coefficients should nevertheless be feasible.

(iii) $m - \log N$ *relation (radio counts).* For the steady-state model, as pointed out in Section 6 (iii), the $m - \log N$ relation for the radio counts will be the same as that for the optical counts provided we make the appropriate interpretation of the symbols. As we have seen in Section 4 (iv), κ^* is very small for the radio spectrum, of order $0{\cdot}2$. Consequently, the coefficient of $\left(\frac{1}{cT}\right) 10^{0{\cdot}2(m-M_0)+1}$ ($\simeq \delta$, when δ is small) in (6.8) is approximately $-1{\cdot}1$.

In the case of the evolutionary models we have to consider (4.4) and (4.5) if the ratio of the spatial densities of radio sources and ordinary galaxies is independent of epoch. If this proviso does not hold then different formulae would be relevant, as exemplified by (5.6) and (5.7). We shall confine our considerations to the former case only.

If we assume that higher time derivatives of parameters are sufficiently small for (4.4) and (4.5) to be a satisfactory fit to the data then the data would provide the coefficient of $\left(\frac{\alpha_1}{c}\right) 10^{0{\cdot}2(m-M_0)+1}$ ($\simeq \delta$, when δ is small) in (4.5), namely $-0{\cdot}3(2 + \kappa^* + \lambda^*)$. Taking $\kappa^* \simeq 0{\cdot}2$, and $\lambda^* < 0$ as in the case of the optical counts, this coefficient would be algebraically greater than $-0{\cdot}7$ in an evolutionary universe, whatever the value of λ^*. Thus, the minimum percentage difference

between these coefficients for the two kinds of universe in the case of radio sources is substantially greater than the difference expected for the optical counts.

As has also been pointed out by McVittie (13), one of the great difficulties in interpreting number count data for radio sources in terms of theoretical formulae is the lack of knowledge of what is to be regarded as the "standard" source of average strength at the present epoch of emission. That is, in terms of the $m-\log N$ formulae occurring in this paper we require to know the absolute magnitude M_0 of the standard radio source. Among the few nearer sources whose distances are known there is unfortunately a wide variation in the strength of emission. It is clear that this makes interpretation of the data in terms of evolutionary trends very uncertain. A reliable working average value for M_0 will not be available until the distances to more sources have been determined.

There is of course the criterion as to whether the gradient of the $m-\log N$ relation steepens or falls from the expected local value of 0·6 as the distance increases. This criterion does not require the determination of M_0. In the case of the steady-state model this gradient must monotonically decrease from 0·6. It is interesting to get some idea of its value for specific value of δ from its exact expression in (6.13). We shall neglect the quantity $dK_0/d\delta$ in this expression since it is approximately equal to κ^* when δ is small, and it is probably always small or at any rate positive. Thus, when $\delta=0{\cdot}1$ we find that $d(\log_{10}N)/dm \simeq 0{\cdot}53$. When $\delta=0{\cdot}2$, near the limit of the optical telescopes, it is nearly 0·46. For radio sources at $\delta=1$ we find that in a steady-state universe the $m-\log_{10}N$ gradient would be as low as 0·25.

On the other hand the results of the Cambridge counts yield a gradient in the region of unity (in our notation), while the Australian counts correspond to a gradient of 0·7. But the marked lack of coincidences between the two surveys with respect to the weaker sources would indicate that serious errors are present in the data, rather than the conclusion that the steady-state theory is not the true cosmology.

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M.N. **119**, No. 3, pp. 213–222, 1959:

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p. 215, penultimate line of second paragraph,
for 1100 μV *read* 11000 μV.

РЕЗЮМЕ ДОКЛАДОВ ОПУБЛИКОВАННЫХ В No. 5 ВЫПУСКЕ, В ПЕРЕВОДЕ НА РУССКИЙ ЯЗЫК

НОВОЕ ОПРЕДЕЛЕНИЕ ИЗМЕНЕНИЯ „ЦЕНТР-КРАЙ" ДЛИН ВОЛН В СПЕКТРЕ СОЛНЦА

М. Г. Адам

Увеличение длины волны от центра к краю, которое наблюдается для солнечных линий средней интенсивности, происходит почти целиком на последних 10% пути по диску. До сих пор не было сделано измерений в этой области с достаточно высоким разрешением диска. 35-м оксфордский телескоп, который дает изображение Солнца радиусом 165 мм, использовался для проведения детальных наблюдений краевого эффекта в 261 точке диска на широтах, близких к солнечному экватору. Все наблюдавшиеся точки лежат близко к краю на расстояниях от центра 85-100% радиуса диска. Отдельные наблюдения показывают относительно средней кривой значительные флуктуации, которые следует приписывать влиянию локальных полей скоростей, существующих, как известно, на Солнце. Показано, что систематические разности, которые выявляются в измерениях на восточном и западном краях для расстояний от центра, больших 98% солнечного радиуса, являются результатом рассеяния света. При определении из наблюдений кривой краевого эффекта крайне важно, чтобы были учтены искажения, обусловленные полями скоростей и рассеянным светом.

Новая кривая краевого эффекта находится в достаточно хорошем согласии со старыми определениями для всей области перекрытия наблюдений. На самом краю имеется сдвиг по отношению к длинам волн для центра дисла, который, если его выразить как доплеровскую скорость, составляет 0,53 км/сек. Это указывает на наличие на самом краю красного смещения в 0,84 км/сек, сравнимого с Эйнштейновским смещением в 0,636 км/сек.

ОБ ИНТЕРПРЕТАЦИИ СОЛНЕЧНОЙ ГРАНУЛЯЦИИ

П. Фелгетт

Методика интерпретации наблюдений солнечной грануляции анализируется со специальным учетом стохастической природы этого физического явления и отсутствия у любого оптического изображения достаточной информации, чтобы позволить вывести в явной форме распределение интенсивности в данном объекте. Многие из отмеченных нами вопросов являются элементарными и по существу не новыми, однако необходимость специального обсуждения и иллюстрации каждого вопроса была доказана появлением за последнее время в литературе результатов, которые не имеют той значимости, которая им приписывается. В качестве применения рассматриваются одновременные наблюдения скоростей и яркости гранул Ричардсоном совместно с Шварцшильдом и Пласкеттом. Делается вывод, что свидетельство в пользу существования корреляции между скоростью и яркостью слабее, чем предполагалось. Гармонически связанные периодичности в двух рядах оксфордских наблюдений, возможно, могут быть связаны с магнитогидродинамическими эффектами, но предположение некоторых авторов о том, что значительные периодичности имеют место над большими областями Солнца, не подтверждается. Данное исследование было запланировано с целью содействия приведению методов интерпретации в большее согласие с современными достижениями в области наблюдений грануляции, а программы, разработанные для выполнения требуемых вычислений на вычислительной машине, EDSAC II, могут послужить основой для быстрой обработки наблюдений в будущем.

ПО ПОВОДУ *H*-ФУНКЦИЙ ДЛЯ ИЗОТРОПНОГО РАССЕЯНИЯ

Д. У. Стибс и Р. Уэйр

Показано, что H-функции для изотропного рассеяния в плоско-параллельной полубесконечной атмосфере могут быть легко и точно вычислены при помощи простой квадратуры. Описываются методы, которые позволяют сохранить точность вычислений в окрестности от особенностей подынтегральной функции и ее производной. Окончательное выражение было программировано для вычислений с вычислительной машиной IBM 704 и проведены вычисления для 160 H-функций для значений альбедо:

$$\varpi = 0\,(0{\cdot}01)\,0{\cdot}90\,(0{\cdot}005)\,0{\cdot}95\,(0{\cdot}001)\,0{\cdot}99\,(0{\cdot}0005)\,1{\cdot}0.$$

Получено хорошее согласие с контрольными вычислениями, включающими моменты нулевого, первого и второго порядков. H-функция, полученная для консервативного случая $\varpi=1$ совпадает с точностью до одной единицы шестого знака со значениями, полученными Плачеком при помощи видоизмененной формы интеграла Винера-Хопфа.

Приводятся таблицы значений H-функций для $\mu = 0\,(0{\cdot}05)\,1{\cdot}0$ и для избранных значений ϖ; табличные значения округлены до единицы шестого знака. Для нахождения H-функций для любого значкния ϖ все множество вычисленных функций было апроксимировано для $\mu = 0\,(0{\cdot}05)\,1{\cdot}0$ при помощи полиномов Чебышева по степеням ϖ и метода наименьших квадратов, причем весь промежуток изменения ϖ от 0 до 1 был разбит на несколько перекрывающих его частей. H-функции, определенные при помощи этих полиномов, обладают точностью до одной единицы четвертого знака. При помощи полиномов представлены также моменты первого порядка.

ДВОЙНОЙ КАРЛИК HD 16157. ПРЕДВАРИТЕЛЬНЫЙ ОТЧЕТ

Д. С. Эванс

Спектроскопические наблюдения этой замечательной звезды проводились с 1951 года при помощи инструментов обсерватории Радклифф, Претория. В настоящее время представляется достаточно достоверным, что это—спектрально-двойная звезда, главной компонент которой является карликом позднего класса K, имеющим орбитальный период 1,56145 суток и очень малую величину функции массы, равную 0,00871 солнечной массы. Предполагается, что на нескольких пластинках найдены признаки спектра второго компонента. Измеренный параллакс оказался 0",083. Спектры похожи на спектр звезды YY Gem в том отношении, что Бальмеровские линии и линии кальция являются эмиссионными. Звезда похожа на YY Gem еще и в том отношении, что она также является переменной, обнаруживающей изменения кривой блеска от одного сезона к другому; эти изменения выражены даже резче, чем у YY Gem. Представляется затруднительным объяснить их на основе обычной модели затменной двойной и потому просто приводятся все полученные до сих пор данные с целью стимулирования независимых наблюдений в других местах.

ВИЗУАЛЬНЫЕ И ДАЛЕКИЕ КРАСНЫЕ ГРАДИЕНТЫ И ЦВЕТОВЫЕ ТЕМПЕРАТУРЫ γ КАССИОПЕИ. II

Д. Р. Барбер

Приводятся дальнейшие ряды спектрофотометрических градиентов γ Cas для средних длин волн 5120 A и 7090 A и соответствующие цветовые температуы для периода от апреля 1951 до декабря 1958 г. Вероятные ошибки значений ϕ составляют $\pm 0{,}03$ (визуальный) и $\pm 0{,}11$ (далекий красный) соответственно.

Хотя в течение шестилетнего интервала, 1952–57, ϕ_{5120} колебалось между крайними значениями 0,56 и 1,23, средний уровень не обнаружил заметных изменений ($\Delta\phi = +0{,}03$) по сравнению с его средним значением для предыдущего шестилетнего периода.

В отличие от этого среднее значение ϕ_{7090} для того же периода, 1952–57, уменьшилось на 0,38, а в конце 1951 г начались циклические колебания далекого красного градиента, амплитуда которых была $\sim 0{,}75$. При нанесении на график эти значения градиента ложатся на кривую, указывающую на период в $\sim 1{,}0$ в 1951–52 и $\sim 1{,}8$ в 1956–57 г. Эти несомненно реальные изменения, по-видимому, связаны с короткопериодическими вариациями инфракрасной интенсивности в спектре γ Cas, обусловленными изменениями непрерывного спектра в области, лежащей в красную сторону от предела серии Пашена.

ШАРОВЫЕ СКОПЛЕНИЯ. Ч.II. СПЕКТРАЛЬНЫЕ ТИПЫ ОТДЕЛЬНЫХ ЗВЕЗД И ТИП ИНТЕГРАЛЬНОГО СПЕКТРА

Т. Д. Кинман

Обсуждается проблема классификации спектров отдельных звезд и интегрального спектра скопления. Найдены типы спектров обоих видов, которые были предварительно рассмотрены в Ч.I, а результаты сравнены с результатами классификаций Мейалла, а также Моргана, основанных на спектрах с меньшей дисперсией. Спектральные типы Мейалла обнаруживают систематические отличия как от Радклиффских типов, так и от типов Моргана. Ввиду этого в спектральные типы Мейалла внесены поправки с целью получения типов интегральных спектров для 63 скоплений в однородной системе.

Обсуждается вопрос о том, как сказывается отношение металл/водород на силах линий металлов в спектрах принадлежащих к скоплениям гигантов, и сделана попытка вычислить величину этого отношения для различных скоплений на основании имеющихся спектров. Сделан вывод, что отношение металл/водород принимает для различных скоплений значения от более 1/10 до менее 1/100 соответствующего значения для Солнца. Для такого скопления, как NGC 5272, показано, что если принять значение этого отношения $\sim 1/10$ значения для Солнца, то спектры принадлежащих скоплению гигантов оказываются почти нормальными для их показателей цвета и светимостей в согласии с наблюдениями, однако линии металлов в спектрах субгигантов должны были бы быть тогда слабее Сделана

попытка предсказать типы интегральных спектров NGC 5272 и (с меньшей уверенностью) пяти других скоплений. Различия в интегральных спектрах вызываются в основном различиями в значениях отношения металл/водород и различием относительной населенности горизонтальной ветви. Предсказанные спектральные типы в пределах ожидаемой точности согласуются с наблюденными, что служит подтверждением правильности нашего анализа.

Найдена корреляция между типом интегрального спектра скопления и концентрацией к галактической плоскости. Скопления самого раннего спектрального типа, звезды в которых имеют малое относительное содержание металлов, образуют обширное сферическое облако с малой концентрацией к этой плоскости; в то же время скопления самого позднего типа, звезды в которых имеют наибольшее содержание металлов, показывают сильную концентрацию к галактической плоскости. Скопления промежуточного типа, которые являются наиболее многочисленными, имеют пространственное распределение промежуточного типа. Также обсуждается менее выраженная корреляция между спектральным типом и свойствами звезд типа RR Лиры.

ШАРОВЫЕ СКОПЛЕНИЯ. Ч.III. АНАЛИЗ ЛУЧЕВЫХ СКОРОСТЕЙ СКОПЛЕНИЙ

Т. Д. Кинман

Определенная по лучевым скоростям 70 шаровых скоплений солнечная скорость равна 167 ± 30 км/сек, и ее апекс не отклоняется от направления галактического вращения Солнца. Обнаружено небольшое, едва ли заслуживающее внимание, видимое изотропное сжатие системы скоплений, которое может быть обусловлено систематическими ошибками скоростей, определенных по спектрам с малой дисперсией. Лучевые скорости не позволяют установить наличие каких-либо дифференциальных движений внутри системы скоплений ни как функции галактоцентрического расстояния, ни как функции интегрального спектрального типа. Однако большие ошибки неизбежно связаны с этими решениями вследствие малочисленности имеющейся статистической выборки скоплений, их неблагоприятного распределения в галактике и больших пекулярных скоростей.

Показано, что звезды поля типа RR Лиры, обладающие теми же периодами, что и звезды этого типа, принадлежащие скоплениям, имеют ту же самую скорость Солнца и дисперсию скоростей, что и сами скопления. Высказано предположение, что эти звезды имеют общее происхождение со скоплениями.

Диаграмма зависимости отношения лучевой скорости к круговой скорости от угла между направлениями от скопления к Солнцу и галактическому центру подтверждает заключение Хернера о том, что орбиты скоплений в большинстве случаев имеют большой эксцентриситет. Большое значение эксцентриситета ($e=0{\cdot}8$) получено также из рассмотрения среднего галактического вращения системы скоплений.

Применена простая схема для нахождения распределения массы в галактике по скоростям скоплений. Результаты находятся в согласии с моделью Шмидта, за исключением, возможно, галактоцентрических расстояний, превышающих 12 кпс.

Показано, что отнесенный к единице массы средний кинетический момент системы скоплений и всей галактики одинаков. Кратко обсуждается значение этого факта для проблемы происхождения системы скоплений.

НЕКОТОРЫЕ ФОРМУЛЫ, СВЯЗЫВАЮШИЕ ГАРВАРДСКИЕ КООРДИНАТЫ x И y, ЭКВАТОРИАЛЬНЫЕ КООРДИНАТЫ И СТАНДАРТНЫЕ КООРДИНАТЫ ДЛЯ МАГЕЛЛАНОВЫХ ОБЛАКОВ

А. Дж. Весселинк

В статье рассматривается чисто практическая задача отождествления звезд в Магеллановых Облаках. Даются формулы, которые переводят гарвардские координаты x и y, экваториальные координаты и стандартные координаты друг в друге. Описывается их использование для отождествления объектов и для нанесения системы координат на снимки. Для того чтобы упростить громадную работу, которая, вероятно, будет затрачена на изучение Магеллановых Облаков, указываются преимущества системы стандартных координат перед гарвардской системой x и y.

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Т. Д. Кинман

Определенная по лучевым скоростям 70 шаровых скоплений солнечная скорость равна 167 ± 30 км/сек, и ее апекс не отклоняется от направления галактического вращения Солнца. Обнаружено небольшое, едва ли заслуживающее внимание, видимое изотропное сжатие системы скоплений, которое может быть обусловлено систематическими ошибками скоростей, определенных по спектрам с малой дисперсией. Лучевые скорости не позволяют установить наличие каких-либо дифференциальных движений внутри системы скоплений ни как функции галактоцентрического расстояния, ни как функции интегрального спектрального типа. Однако большие ошибки неизбежно связаны с этими решениями вследствие малочисленности имеющейся статистической выборки скоплений, их неблагоприятного распределения в галактике и больших пекулярных скоростей.

Показано, что звезды поля типа RR Лиры, обладающие теми же периодами, что и звезды этого типа, принадлежащие скоплениям, имеют ту же самую скорость Солнца и дисперсию скоростей, что и сами скопления. Высказано предположение, что эти звезды имеют общее происхождение со скоплениями.

Диаграмма зависимости отношения лучевой скорости к круговой скорости от угла между направлениями от скопления к Солнцу и галактическому центру подтверждает заключение Хернера о том, что орбиты скоплений в большинстве случаев имеют большой эксцентриситет. Большое значение эксцентриситета ($e = 0{\cdot}8$) получено также из рассмотрения среднего галактического вращения системы скоплений.

Применена простая схема для нахождения распределения массы в галактике по скоростям скоплений. Результаты находятся в согласии с моделью Шмидта, за исключением, возможно, галактоцентрических расстояний, превышающих 12 кпс.

Показано, что отнесенный к единице массы средний кинетический момент системы скоплений и всей галактики одинаков. Кратко обсуждается значение этого факта для проблемы происхождения системы скоплений.

НЕКОТОРЫЕ ФОРМУЛЫ, СВЯЗЫВАЮЩИЕ ГАРВАРДСКИЕ КООРДИНАТЫ x И y, ЭКВАТОРИАЛЬНЫЕ КООРДИНАТЫ И СТАНДАРТНЫЕ КООРДИНАТЫ ДЛЯ МАГЕЛЛАНОВЫХ ОБЛАКОВ

А. Дж. Весселинк

В статье рассматривается чисто практическая задача отождествления звезд в Магеллановых Облаках. Даются формулы, которые переводят гарвардские координаты x и y, экваториальные координаты и стандартные координаты друг в друге. Описывается их использование для отождествления объектов и для нанесения системы координат на снимки. Для того чтобы упростить громадную работу, которая, вероятно, будет затрачена на изучение Магеллановых Облаков, указываются преимущества системы стандартных координат перед гарвардской системой x и y.

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CONTENTS

PAGE

Meeting of 1959 October 9:
- Death of William Hammond Wright 585
- Fellows and Junior Members elected 585
- Presents announced 585

Meeting of 1959 November 13:
- Fellows and Junior Members elected 586
- Presents announced 586

Meeting of 1959 December 11:
- Death of William Moody Witchell 587
- Fellows elected 587
- Junior Members elected 592
- Presents announced 592

A. A. Mikhailov, The deflection of light by the gravitational field of the Sun (George Darwin Lecture)... 593

B. E. J. Pagel, Note on collisional dissociation of the H^- ion in the solar atmosphere 609

C. A. Murray, On the determination of ephemeris time 612

J. E. Geake, J. Ring and N. J. Woolf, The Fabry–Perot monochromator 616

A. D. Thackeray, The emission line spectrum of the eclipsing variable AR Pavonis 629

David S. Evans, A. Menzies and R. H. Stoy, Fundamental data for southern stars (Second List) 638

C. Hazard and D. Walsh, An experimental investigation of the effects of confusion in a survey of localized radio sources 648

A. W. K. Metzner and P. Morrison, The flow of information in cosmological models 657

W. Davidson, Number count relations in observational cosmology... 665

Erratum 681

Russian summaries of papers in *M.N.*, **119,** No. 5 682

РЕЗЮМЕ ДОКЛАДОВ ОПУБЛИКОВАННЫХ В No. 5 ВЫПУСКЕ, В ПЕРЕВОДЕ НА РУССКИЙ ЯЗЫК 682

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